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Information Economics

History

As recalled by Laffont and Martimort (2002), for centuries, farmers have labored the fields belonging to their landlords, apprentices have worked under the orders of their master craftsman. The motivations behind these behaviors are not obvious to guess. For long, authority was seen as the major explanation but with the development of trade and industry, the economic motivation came to play a role. The theoretical debate is launched by Smith (1776) who criticizes sharecropping, the metayage system, where the farmer and the landlord share evenly the harvest because it acts like a tax that inefficiently reduces the incentive of the farmer to apply labour on the land. Hence, whenever the landlord can not monitor the work of the farmer to enforce the adequate amount of effort, we have a clear example of what is now called “moral hazard” because the farmer will shirk or cultivate his personal vegetable garden.

Task delegation occurs because the landlord has a limited time and ability but above all because the division of labour and the specialization enable great economies of scale. Given the complexity of most productive activities today, delegation goes along with the loss of supervision, the inability for the landlord to monitor or control the activities of the farmer. Being left alone performing his task, the farmer learns more than the landlord about all economic aspects of the productive activity he is involved in; he acquires some private information. Furthermore, the very existence of delegation, means that the farmer is more or less able to orient the activity as he wishes.

The modern theory calls principals those who pay, agents those who receive payment in exchange for their activity. To ease exposition, we systemically refer to the principal as “she” and to the agent as “he”. Their relationship form an agency and whenever one party holds (relevant) private information, there is asymmetric information. The landlord–farmer case is only one in many situations fitting the formalization as shown in the table below.
Table 1: Examples of Agency Relationships

<table>
<thead>
<tr>
<th>Principal</th>
<th>Agent</th>
<th>Activity</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>landlord</td>
<td>farmer</td>
<td>cultivation</td>
<td>weather</td>
</tr>
<tr>
<td>client</td>
<td>attorney</td>
<td>defense</td>
<td>relation of the case to law</td>
</tr>
<tr>
<td>saver</td>
<td>broker</td>
<td>investment</td>
<td>market opportunities</td>
</tr>
<tr>
<td>stockholder</td>
<td>manager</td>
<td>industrial policy</td>
<td>market conditions</td>
</tr>
<tr>
<td>government</td>
<td>builder</td>
<td>public work</td>
<td>cost</td>
</tr>
<tr>
<td>city</td>
<td>utility</td>
<td>water</td>
<td>technology</td>
</tr>
<tr>
<td>insurer</td>
<td>policyholder</td>
<td>health</td>
<td>genetic risk</td>
</tr>
</tbody>
</table>

Typology

It is only recently that economists have come to realize that the private information held by the agent is crucial to determine the efficient decision, the one maximizing the economic surplus of the agency relation.\(^1\) To make sure that this optimal action will be carried out by the agent, the principal must elicit this information and then convince the agent to take that action. The latter part is by no means trivial; as we can check from the examples in the above table, the principal can hardly control the effort (e.g., time, money) expanded by the agent in the activity he is getting paid for. From an analytical point of view we are facing a problem of *hidden information* and/or *hidden action*. The usual terminology nevertheless follows the insurance vocabulary because the first theoretical works were developed in that field.

The classical case of hidden information is when risky people try to get maximum coverage; insurers speak of an *adverse selection* because all premiums will have to be raised and may discourage those at less risk from insuring themselves. Akerlof (1970) introduces the term *Adverse Selection* when citing a 1964 textbook on insurance “There is potential adverse selection in the fact that healthy term insurance policy holders may decide to terminate their coverage when they become older and premiums mount. This action could leave an insurer with an undue proportion of below average risks and claims might be higher than anticipated.”

The typical circumstance for hidden action is when drivers could drive with care to avoid accidents but fail to do so once they are fully insured; this behavior

\(^1\)An early modern precursor is Leibenstein (1966) with the concept of *X-efficiency* seen in §§??.
is known as *moral hazard*. Health insurance professionals defined moral hazard in the 1960s as the “intangible loss-producing propensity of the insured individual” or the “hazard that arises from the failure of individuals who are affected by insurance to uphold the accepted moral qualities”. Pauly (1968) accurately showed that this was no morality problem but a simple consequence of rational economic behavior: by spreading the cost of my health-care over the entire population (socializing), medical insurance makes this service cheaper to me, thus my demand for it increases.
Chapter 1

Risk and Uncertainty

Human have always faced and feared risk; many practices adopted by primitive tribes can be seen as insurance mechanisms. In this chapter, we review quickly how one models risk and uncertainty with probabilities and how the standard theory of demand can be extended to account for randomness. We then introduce a basic measure of risk and characterize the way firms and individuals adjust their behavior when exposed to risk. We end with some more advanced results that are useful for the following chapters.
1.1 Choice under Uncertainty

Introduction

The concept of asymmetric information builds on the more general idea of incomplete information, the fact that economic agents ignore some relevant features and are therefore faced with uncertainty and exposed to risk. Let us use a simple example derived from our typical oligopoly framework: when a manager fails to observe the marketing strategy of his competitor, he lacks a crucial information and therefore faces risk; his strategy will have to account for this uncertainty. Although the market outcome is a deterministic function of the two strategies used by firm A and B, it is random from the manager A’s point of view. Take for instance the market equation of the Cournot model \( p = 1 - q_A - q_B \). If manager A is unsure of what quantity \( q_B \) was decided upon by manager B, then the price becomes uncertain since his own decision \( q_A \) does not determine completely the final price (it only limits the range of possibilities).

To illustrate the importance and ubiquity of risk and uncertainty, we reproduce in Table 1.1, a computation done by the US Energy Information Administration for a variety of important commodities. Volatility is a statistical measure of how often the observed price is far away from its mean i.e., it is an assessment of the lack of regularity of the commodity’s price.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Electricity</th>
<th>Gas</th>
<th>Petroleum</th>
<th>S&amp;P 500</th>
<th>Treasury Bonds</th>
<th>Copper</th>
<th>Gold</th>
<th>Coffee</th>
<th>Sugar</th>
<th>Corn</th>
<th>Cotton</th>
<th>Pork</th>
<th>Beef</th>
<th>Cattle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility (%)</td>
<td>300</td>
<td>80</td>
<td>40</td>
<td>15</td>
<td>13</td>
<td>32</td>
<td>12</td>
<td>40</td>
<td>100</td>
<td>40</td>
<td>80</td>
<td>72</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Spot Market Prices Annual Volatility

To cope with risk, most people buy insurance e.g., life, car or housing insurance or pension plans; the importance of the insurance sector in developed economies attests of this general desire to diminish risk. To assess how the presence of risk, uncertainty and asymmetric information impinges on economic choices, it is necessary to extend the standard theory of demand.
1.1.1 Time and Money

Because uncertainty relates to the future and “time is money”, we have to account for the opportunity cost of time. We can then build the net present value (NPV) to aggregate a stream of revenue (or disbursment). This concept builds on the subjective preference for present consumption also called time preference of the investor.

Once we make the mental experiment of viewing today’s good as different from tomorrow’s, we can transpose the intertemporal allocation of one good between periods into the standard microeconomic analysis of choice (cf. §??). Thus, the RMS from today to tomorrow should be equated to the price ratio of money at the successive times. Now, if money could flow freely between periods, then the latter ratio ought to be unity. Yet, in most cases, we require compensation to forgo today’s consumption which means that one unit today is worth more tomorrow. If there is no market involved because the consumer is solving a cake-eating problem (he allocates some amount between the two periods), then the relative price simply expresses his subjective preference for present consumption. This principle valid for two periods readily extend to many using a chain reasoning.

The transposition to finance is straightforward. When an economic agent, whether a consumer or a firm, lends or invests 1€ for a period of time, she forgoes an opportunity for immediate consumption or alternative use; therefore, she requires a compensation in the form of an interest at repayment time i.e., she receives $1 + r$ where $r$ is the discount rate. The market interest rate $r_0$ is an (objective) average of the (subjective) discount rates of all investors.⁠¹ Typically, an individual will lend (resp. borrow) if $r < r_0$ (resp. $>$); if there is an excess of lending or borrowing, the market rate adjusts to restore equilibrium.

The discount factor $\delta \equiv \frac{1}{1+r} < 1$ is the present value (PV) of 1€ to paid within one period of time. The PV of 1€ to paid within $t$ periods of time is $\frac{1}{(1+r)^t}$; it is found by compounding the periodic discount factor. The time period can be a year, month, week, day, hour or minute. The relation between the corresponding rates is found by applying compounding. If $r_a$ is an annual rate, the associated monthly rate $r_m$ is the solution of $(1 + r_m)^{12} = 1 + r_a$ which, in a first approxima-

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⁠¹It is called risk-free because the securities emitted by governments of countries who never failed to pay their obligations are considered to be without risk of default (cf. Sovereign Ratings).
tion can be taken to be $r_m \approx r_a / 12$. It is often convenient to consider time as a continuous variable. Notice from the previous calculation that the “per second” interest rate $r$ is so small (in comparison with the annual one) that the formula $(1 + r)^t = e^{t \ln(1 + r)} \approx e^{rt}$ becomes exact since $\ln(1 + r) \approx r$ for small $r$ as shown by Hotelling (1925).

The NPV of a series of financial (cash) flows $y = (y_0, y_1, y_2, ..., y_T)$ occurring at periodic intervals is then

$$PV(y) \equiv \sum_{t=1}^{T} \frac{y_t}{(1 + r)^t}$$

in the continuous time version.

1.1.2 Probability Theory

Up to now in this book, information was distributed evenly among actors i.e., a relevant piece of information was either known to everyone that might be interested by its contents or known to no-one. The key novelty of this Part on information is that some people have private information i.e., know more about some relevant facts than others. From the point of view of those in the dark, several alternatives are to be considered and thus balanced or weighted. Probability theory is the mathematical tool that allow the extension of economic concept to this larger world.

From now on, any economic phenomena involving uncertainty over a dimension $x$ will give rise to a random variable denoted $\tilde{x}$. Its law is $H(x) = \text{Prob}(\tilde{x} \leq x)$ and its density (if it exists) is $h(x) = H'(x)$. Whatever the real function $f$, the expectation of the random variable $f(\tilde{x})$ is $\mathbb{E}[f(\tilde{x})] = \int f(x) \, dH(x) = \int f(x) \, h(x) \, dx$ (if $h$ exists). The theory of rational choice characterized by the concave increasing utility function of income $u(.)$ is thus amended for the presence of uncertainty by adopting the expected utility hypothesis i.e., replacing $u(x)$ by $\mathbb{E}[u(\tilde{x})]$.

We say that an agent holds information $\theta$ with respect to the underlying phenomenon $\tilde{x}$ if he believes the random phenomenon to be distributed according to the subjective law $H_\theta$; the latter can be interpreted as an updating of the original objective distribution $H$ upon learning $\theta$. The expectation of $f(\tilde{x})$ conditional on the knowledge of $\theta$ is $\mathbb{E}[f(\tilde{x}) | \theta] = \int f(x) \, dH_\theta(x)$ i.e., the objective distribution $H$ is replaced by the subjective one $H_\theta$. 

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1.1.3 Measuring Risk

Insurance vs. Gamble

When consumers display decreasing marginal utility of income, they are risk averse i.e., they dislike any fair gamble (one with zero expectation). This constitutes the main explanation for the widespread demand of insurance. However people have and will continue to bet on sports, buy lottery tickets and invest in speculative shares, all of which are unfair gambles. To reconcile this “risk-loving” gambling behavior with the “risk-averse” insuring one, Friedman and Savage (1948) propose to draw the utility of income as shown on Figure 1.1: it displays risk-aversion (concavity) from zero until some level significantly greater than the current income of the individual (say twice); from then on, the curve displays a risk-loving attitude towards risk (convexity).

As can be checked on Figure 1.1, the consumer prefers the certain income \( x \) to the fair gamble \( \tilde{x} \) that amounts to win or lose \( \delta \) with identical probability (\( \mathbb{E}[\tilde{x}] = x \)). To understand why a lottery ticket might appear attractive, recall that lotteries are gambles with a quasi certain small loss (the ticket price) and a small probability of winning a big prize (\( z \) on Figure 1.1) that dramatically changes life’s opportunities for the better which is why the utility at \( z \) is so large. It is then possible that the expected utility of the lottery ticket draws a point on the chord that lies above the utility curve i.e., it was rational to buy it in the first place. This occurs if either the probability of winning is objectively not too small or subjectively inflated or, and this is the most frequent case, the individual places a very high value on his new life style upon winning the prize. Since risk loving behavior involves psychological elements, we concentrate on risk-aversion which is more amenable to economic treatment (although the two are mathematically symmetrical one from another).

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The mathematical result is Jensen inequality: \( \mathbb{E}[u(\tilde{x})] < u(\mathbb{E}[\tilde{x}]) \). Indeed, the concavity of \( u \) is equivalent to the property \( \frac{u(x)-u(x_0)}{x-x_0} < u'(x_0) \) (the chord is flatter than the tangent at \( x_0 \)); taking \( x_0 = \mathbb{E}[\tilde{x}] \) and integrating we obtain \( \mathbb{E}[u(\tilde{x})] < \mathbb{E}\left[u(x_0) + u'(x_0)(\tilde{x} - x_0)\right] = u(x_0) + u'(x_0)\mathbb{E}[\tilde{x} - x_0] = u(\mathbb{E}[\tilde{x}]) \).
Since utility is intangible (\( u \) is not unique), the positive difference \( u(\mathbb{E}[\tilde{x}]) - \mathbb{E}[u(\tilde{x})] \) we observe on Figure 1.1 has no particular meaning and fail to measure adequately the amount of risk faced by the individual. We therefore develop a monetary value to express it. As we can observe, the random income \( \tilde{x} \) has a certainty equivalent \( y \) solving the equation \( u(y) = \mathbb{E}[u(\tilde{x})] \); the difference \( \mu \equiv \mathbb{E}[\tilde{x}] - y \) is the risk premium that the investor would agree to pay in order to avoid the risk associated with the random income \( \tilde{x} \). This is a subjective value different from an insurance risk premium which is an objective market value.

**Degree of Risk Aversion**

Arrow (1965) and Pratt (1964) define an individual to be more risk averse than another if he refuses all gambles that the former refuses. Defining the index of absolute risk aversion (ARA)\(^3\) \( \rho_u(x) \equiv -\frac{u''(x)}{u'(x)} > 0 \), they show equivalence of the following assertions:

\(^3\)Observe that the function \( \rho \) completely characterizes the preferences since it enables to recover the utility function \( u \) up to an affine transformation.
Agent with utility $v$ is more risk averse than agent with utility $u$.

$v$ is a concave transformation of $u$.

$\rho_v > \rho_u$.

agent $v$'s risk premium is larger than agent $u$'s.

It can also be shown that the risk premium decreases with wealth if and only if the absolute risk aversion is also decreasing with wealth. Next, if the absolute risk aversion $\rho$ is constant then the CARA utility function is $u(x) = -e^{-\rho x}$ (up to an affine transformation). Finally, the relation between the index of risk aversion and the risk premium can be pinpointed if the random income is normally distributed with mean $\mu$ and variance $\sigma^2$ which we write $\tilde{x} \sim \mathcal{N}(\mu, \sigma)$ for short. An important property of the normal law is that $\mathbb{E}[e^{\tilde{x}}] = e^{\mu + \frac{1}{2} \sigma^2}$. Upon observing that $-\rho \tilde{x} \sim \mathcal{N}(-\rho \mu, \rho \sigma)$, we deduce that

$$\mathbb{E}[u(\tilde{x})] = -\mathbb{E}[e^{-\rho \tilde{x}}] = -e^{-\mu \rho + \frac{\rho^2 \sigma^2}{2}} = u\left(\mu - \frac{1}{2} \rho \sigma^2\right)$$

(1.2)

meaning that the risk premium is exactly $\frac{1}{2} \rho \sigma^2$. The index of absolute risk aversion $\rho$ therefore represents twice the “risk premium per unit of variance” (for infinitesimal risk). The previous formula also tells us that an agent with CARA preferences facing risk normally distributed aims to maximize $\mu - \frac{1}{2} \rho \sigma^2$.

An individual displaying constant absolute risk aversion $\rho$ and whose final income is a random normal variable with expectation $\mu$ and standard deviation $\sigma$ act so as to maximize $\mu - \frac{1}{2} \rho \sigma^2$.

### 1.2 Firm Behavior under Risk

**Firm and Risk**

Large profits are highly valued by the stockholders of a firm but they do not necessarily lead to great monetary rewards for the managers or great burst of pride. On the contrary, great losses can lead to destitution or bankruptcy which are very negative outcomes for managers. It is therefore plausible to assume that the representative manager of a firm has a utility function that is increasing with
profits but concave i.e., displaying risk-aversion. On top of maximizing profits, one would expect managers to act so as to reduce their risk exposure. We now explain why this is hardly feasible.

The two classical instruments to eliminate risk are negative correlation (do not put all your eggs in the same basket) and the law of large numbers (try to manage many baskets). Regarding correlation, the very existence of a firm originates in the expertise and know-how of its creator regarding a particular sector of the economy. Its activities take place in the same region or the same markets; thus, they are subject to the same exogenous shocks. In statistical terms, all the revenue generating activities of the firm are positively correlated which means that profits will be highly variable across time. As for the law of large number, non financial firms tend to engage into a limited number of projects that are commensurate with their current size (as measured for instance by liquid assets); this is done to take advantage of scale economies and achieve a high profitability.\(^4\)

**Behavioral Consequences**

Sandmo (1971) studies the consequences of the manager’s risk-aversion upon his productive behavior within the framework of the neoclassical theory of the competitive firm. Firm’s profit is \(\pi = pq - C(q)\) while the manager’s objective is \(u(\pi)\) where the utility function satisfies \(u'' < 0 < u'\). In a risky world, the market price is a random variable \(\tilde{p}\) with mean \(p_0\) that is determined after the production choice \(q\).

The benchmark corresponds to either risk neutrality (\(u\) linear) or absence of risk (constant price); in that case the optimal production is \(q^*\) solving \(p_0 = C_m(q)\).\(^5\) In the general case, the manager maximizes \(\mathbb{E}[u(\tilde{\pi})]\) where \(\mathbb{E}\) denotes the expectation operator corresponding to the manager’s belief regarding the distribution of \(\tilde{p}\). The FOC of maximization in quantity is

\[
0 = \mathbb{E}\left[u'(\tilde{\pi})(\tilde{p} - C_m(q))\right] \Leftrightarrow \mathbb{E}\left[u'(\tilde{\pi})\tilde{p}\right] = \mathbb{E}\left[u'(\tilde{\pi})C_m(q)\right] = C_m(q)\mathbb{E}\left[u'(\tilde{\pi})\right] \quad (1.3)
\]

\(^4\)The previous reasoning is false for a financial firm since it always takes care to diversify its assets across markets and activities in order that a loss somewhere be always compensated by a gain elsewhere; also it tries to build large portfolio of similar clients in order to smooth out the effect of chance (or bad luck) down to the mean of the underlying risk.

\(^5\)We assume that the resulting profit is non negative.
because the marginal cost depends on the deterministic quantity chosen by the manager. Observe now that the covariance of $\tilde{p}$ and $u'(\tilde{\pi}(\tilde{p}))$ is negative because $\pi$ is increasing and $u'$ is decreasing. If so then the expectation of the product is less than the product of expectations: $p_0E[u'(\tilde{\pi})] \geq E[u'(\tilde{\pi})\tilde{p}] = C_m(q)E[u'(\tilde{\pi})]$ by (1.3). We have thus shown that $p_0 > C_m$ at the optimum which proves that the optimal quantity under uncertainty is lesser than the certainty equivalent $q^*$. To conclude

Risk exposure induces a competitive firm to reduce her output.

A more definite conclusion can reached for the case where the manager's risk aversion is constant ($\rho$) and the market price is a normal random variable with mean $p_0$ and variance $\sigma^2$. As we already showed in formula (1.2), the manager then maximizes $U(q) \equiv E[\tilde{\pi}] - \frac{1}{2}\rho \varpi[\tilde{\pi}]$. We already know that $E[\tilde{\pi}] = p_0q - C(q)$ while it is easy to check that $\varpi[\tilde{\pi}] = \sigma^2 q^2$, thus $U(q) = p_0q - C(q) - \frac{1}{2}\rho\sigma^2 q^2$ and the FOC for maximization is

$$p_0 = C_m(q) + \rho\sigma^2$$

as if the real marginal cost was inflated by the risk premium $\rho\sigma^2$.

A last observation is that production will really takes place only if the expected utility $E[u(\tilde{\pi})]$ is greater than $u(-F)$, the utility level in case of zero production. Now, since $u$ is concave, Jensen's inequality (cf. footnote 2) tells us that $E[u(\tilde{\pi})] < u(E[\tilde{\pi}]) = u(\bar{\pi})$ thus, $\bar{\pi} > -F \Rightarrow p_0 > \frac{C(q)}{q}$. We conclude that

A competitive risk averse firm to enter a risky market only if it obtains some economic or extraordinary profits which can be interpreted as a required risk premium.

---

6If $Y = f(X)$ where $f' < 0$, then $0 \geq (X - X_0)(Y - Y_0)$ whether $X < (> )X_0$ because the second term is always of the opposite sign of the first. Integrating conserves the inequality.
1.3 Advanced Topics

Optimal Amount of Risk

We would like to know how a consumer reacts to changes in wealth and what distinguish the behavior of people with different risk attitudes. For instance, what is the optimal amount of risk for a risk averse agent when building his portfolio? Shall a very risk averse person avoid all forms of risk? Contrarily to intuition, Arrow (1965) answers negatively whatever the riskiness and the risk aversion. Indeed, everyone is risk neutral with respect to very small changes and the only thing that matters in that case is the expected return. Hence, anyone will invest at least a small amount into a risky asset which is, on average, more profitable than a risk-less asset such as a treasury bond.

To prove formally this claim, consider investing $1\in\$\ into a combination of a risky asset whose return is the random variable $\tilde{r}$ and the risk-free asset whose sure return is $r_0$. We ought to show that the optimal share $\lambda$ of risky asset is positive. Observe that the final wealth of the individual is $\tilde{w} = (1-\lambda)(1+r_0) + \lambda(1+\tilde{r}) = 1 + r_0 + \lambda(\tilde{r} - r_0)$. Letting $H$ denote the law of $\tilde{r}$, the expected utility is

$$U(\lambda) \equiv \mathbb{E}[u(\tilde{w})] = \int u(1 + r_0 + \lambda(r - r_0)) dH(r)$$

The FOC of maximization is

$$0 = U'(\lambda) = \int (r - r_0)u'(1 + r_0 + \lambda(r - r_0)) (r) dH(r)$$

and since $U'(0) = u'(1 + r_0) \int (r - r_0) dH(r) = u'(1 + r_0) (\mathbb{E}[\tilde{r}] - r_0)$, there is an incentive to buy some of the risky asset ($\lambda > 0$) as soon as the risky asset is on average more profitable than the risk-less one.

Arrow (1965) also demonstrates that a less risk averse person invests more into the risky asset. Intuition would suggest that richer people take on more risk; this is correct for an agent with decreasing absolute risk aversion (DARA) since becoming richer turns him into a less risk averse person (and we can apply the previous result).
Mixing Action and Chance

When an action deterministically generates an outcome, the simple observation
of the later enables to infer with exactitude the former. An example is cultivation
inside a greenhouse: yield $q$ is directly related to the effort $e$ expanded by the
farmer by a relation such as $q = \sqrt{3}e$. Upon observing $q$, we infer that effort was
$e = \frac{q^2}{3}$.

Outdoor cultivation, on the other hand, is subject to weather variations so
that a high yield can either reflect fair conditions or hard work and similarly,
a poor yield can either reflect laziness or the losses due to a storm. Generally
speaking, the action (input) is inaccurately reflected by the result (output). A
simple formalization would be $\tilde{q} = e + \tilde{z}$ where $e$ is the effort, amount of daily
time spend on the field while the random variable $\tilde{z}$ captures the effect of weather
variability; as a consequence, yield is also a random variable $\tilde{q}$.

To capture the interaction of effort and chance, we write $H(q | e) = Pr(\tilde{q} \leq q | e)$ the probability to observe a result lesser than $q$ given that action $e$ was
taken. The expected value of $\tilde{q}$ conditional on the action $e$ is then written $E[\tilde{q} | e] = \int q h(q | e) dq$ where $h(\cdot | e)$ is the density associated to the distribution $H(\cdot | e)$. Likewise the expectation of the random variable $f(\tilde{q})$ is denoted $E[f(\tilde{q}) | e]$.

The previous modeling was adapted to moral hazard where the action of the
decision maker, called the agent, is hidden to another party, called the principal.
In problems of adverse selection, it is a piece of information known to the agent
that is hidden from the principal. In that case, the notation is $\theta$ instead of $e$.

Comparing Risks

An interesting development of the risk concept is the objective comparisons of
risky gambles i.e., rankings that are independent of the players. A gamble differs
from another one if something was done differently i.e., one follows an action $\theta$
and the other one follows a different action $\hat{\theta}$. We can thus call them gambles
$\theta$ and $\hat{\theta}$. We say that there is first order stochastic dominance (FSD) of the distri-
bution $H(\cdot | \hat{\theta})$ over $H(\cdot | \theta)$ if $\forall x \geq 0, H(x | \hat{\theta}) \leq H(x | \theta)$. This property simply
means that a large result is more probable under action $\hat{\theta}$ than under $\theta$. It is
then rather simple to show that anyone prefers gamble $\hat{\theta}$ over $\theta$ since whatever
his (increasing) utility function $u$, we have $E[u(\tilde{x}) | \hat{\theta}] \geq E[u(\tilde{x}) | \theta]$. 17
Although very powerful, this kind of comparison among risks is not very useful because few distributions can be ranked according to FSD. It is therefore natural to ask if the addition of risk-aversion (\( u \) concave) enables to weaken the conditions to compare gambles. The answer is positive and consists in introducing second order stochastic dominance (SSD) of \( H(.|\hat{\theta}) \) over \( H(.|\theta) \) as \( \forall x \geq 0, \int_0^x H(t|\hat{\theta}) \, dt \leq \int_0^x H(t|\theta) \, dt \). As shown by Rothschild and Stiglitz (1970), \( \mathbb{E}[u(\tilde{x})|\hat{\theta}] \geq \mathbb{E}[u(\tilde{x})|\theta] \) for any concave increasing \( u \) if and only if there is SSD of \( H(.|\hat{\theta}) \) over \( H(.|\theta) \).

**Informativeness**

In matters of asymmetric information, *inference* is fundamental and can be presented as follows: “given the observation of result \( x \), what is the probability that the action undertaken was \( \theta \)?”; it will obviously depend on the initial belief we hold regarding the unknown action.

Milgrom (1981) defines result \( x \) to be *more informative* than result \( y \) about a higher action if the observer believes higher actions more probable upon observing \( x \) than upon observing \( y \). This notion of good news can be related to a property pervasive in information economics: the action change from \( \theta \) to \( \hat{\theta} \) satisfies the *monotone likelihood ratio property* (MLRP) if

\[
\hat{\theta} > \theta \Rightarrow \frac{h(x|\hat{\theta})}{h(x|\theta)} \uparrow \text{ in } x \tag{1.5}
\]

The family of distributions \( h(.|\theta) \) is said to satisfy MLRP if (1.5) is true for all parameter values. It is noticeable that MLRP is stronger than FSD i.e., \( \hat{\theta} > \theta \Rightarrow H(.|\hat{\theta}) \leq H(.|\theta) \). To understand the MLRP concept, imagine that the likelihood of observing result \( y \) is the same after the two actions i.e., \( h(y|\hat{\theta}) = h(y|\theta) \), then result \( x > y \) is more likely to appear after the higher action i.e., \( h(x|\hat{\theta}) > h(x|\theta) \).

When a change towards a higher action satisfies the MLRP, greater outputs are signals of greater inputs (but not a proof).

To apply the MLRP concept of informativeness, we define a function \( f \) to satisfy the *single-crossing property* (SCP) if it crosses the axis only once and from
below i.e.,
\[ \exists y, \forall x, (x - y) f(x) \geq 0 \quad (1.6) \]

A fundamental theorem follows: if the family of distributions \( h(. | \theta) \) satisfies the MLRP and \( f \) satisfies SCP, then
\[
\hat{\theta} > \theta \implies \mathbb{E}[f(\tilde{x}) | \hat{\theta}] > \mathbb{E}[f(\tilde{x}) | \theta] \quad (1.7)
\]

Another characterization is that a change in distribution of the risky asset increases its demand whatever the risk aversion (\( u \)) and whatever the risk-free rate (\( r_0 \)) if and only if the change satisfies the MLRP.
Chapter 2

Moral Hazard

One of the oldest contractual relationship in agriculture is tenancy. Under fixed rent, the farmer pays the landlord a monetary rent every year for using of land while under sharecropping he shares the crop with the landlord; alternatively, the farmer can become a laborer to earn a fixed wage. The latter formula works well when parties work in team but if the laborer is left without monitoring, he will shirk and the yield will be very low. At the other extreme, the fixed rent motivates the farmer to exploit optimally the fields because he gets to keep all the crop; the landlord can thus ask a high rent because on average the yield will be high. Everything’s fine when the weather is good but if the winter is very cold or the summer very dry, the yield might severely drop and leave the farmer without enough seeds to replant, feed his family and pay the rent at the same time i.e, agriculture is a risky activity. This may well be the reason why some people prefer to be laborer than farmers.

The landlord faces a trade-off when dealing with the farmer: incentives vs. insurance. Sharecropping therefore appears as a solution mixing both features. On the one hand, any additional effort expanded by the farmer will generate a higher yield (on average), half of which goes into his pocket; this is a good motivation for hard work. On the other hand, the farmer does not have to pay a monetary rent; he is less at risk in case of a bad harvest. The landlord is now sharing risk with the farmer.

The plan of the chapter is as follows. We first detail the agency relationship, the basic incentive problem it faces and various remedies to it. We then use managerial incentives to introduce asymmetric information and the resulting inefficiency. The next section is more formal and emphasizes unexpected
contingencies. The last section presents some extensions regarding the possible renegotiation of contracts. We use material borrowed from Rasmusen (2006), Macho-Stadler and Pérez-Castrillo (1996) and Boccard (2002).

2.1 The Agency Relationship

Profit maximization calls for the pursuit of many goals such as cost minimization, quality enhancement or good customer relationships. Consider then a manager in charge of one such activity; the attainment of the goal requires work dedication, effort and a personal investment. To induce the manager to perform a significant effort, the owner can link a wage bonus to the attainment of an (explicit) objective. Now, to generate an additional improvement, an additional effort is required which can be implemented by increasing the bonus.

2.1.1 Framework

A fundamental assumption underlying the contract literature is that economic relations take place within a well defined and smoothly functioning legal framework. When economic agents sign a contract, they are bound to respect its terms whenever a court of law is able to understand them. This has two consequences; on the one hand, the contract can only include clauses and obligations that are verifiable by the judge or arbitrator but on the other hand, there cannot be any haggling over a verifiable item.¹

Preferences

The principal (e.g., landlord) hires an agent (e.g., farmer) to exert an effort $e$ (e.g., daily hours of work). Applied into the production technology, this input gives rise to an output of value $q(e)$ (e.g., crop) where $q(.)$ is increasing and concave i.e., we assume decreasing returns to scale. The profit for the principal is $\pi \equiv q(e) - w$ where the wage $w$, paid to the agent, can be contingent on verifiable events. To simplify the analysis, we assume that the preferences of the agent for

¹There is a subtle difference in the literature regarding the action of the agent: it can be observable by the principal only or verifiable by the judge (on top of being observable); this distinction is sometimes referred to as soft vs. hard evidence.
effort and income are separable with \( U(w, e) = u(w) - c(e) \) where \( u \) is increasing concave (decreasing returns to wealth) and \( c \) is increasing convex (increasing value of forgone leisure time). This specification has the great advantage that money can be transferred between the principal and the agent to reach any participation constraint. We normalize the utility function so that \( u(0) - c(0) = 0 \).

**Verifiable effort**

When the effort can be contracted upon, a simple contract is \((\hat{e}, \hat{w})\) meaning “do at least \( \hat{e} \) and you’ll get \( \hat{w} \) (otherwise nothing)”. Recalling, that the moral hazard situation refers to the fact that effort is undertaken by the agent, the latter can either choose some \( e \geq \hat{e} \) and derive utility \( u(\hat{w}) - c(e) \) or choose some \( e < \hat{e} \) and derive utility \( u(0) - c(e) \). His best options under the two broad alternatives are thus \( \hat{e} \) and 0, as he dislikes effort. The principal will succeed with his objective of having the agent expand effort \( \hat{e} \) if \( u(\hat{w}) - c(\hat{e}) \geq u(0) - c(0) = 0 \). A first result is thus the obvious observation that the salary must compensate the agent for the toil of working.

**Individual Rationality**

To simplify matters we assume that a single principal is facing a multitude of potential agents to whom she makes a “take-it-or-leave-it” offer. This extreme formulation is justified by the excess demand for the position that turns potential agents into Bertrand competitors ready to accept no more than their opportunity cost \( u \) to get the job; one can think of \( u \) as a minimum wage below which agents prefer to stay at home. This setting apparently opposes an all-mighty capitalist to an harmless worker; this is only done to ease the mathematical analysis. As we shall see later on, the opportunity cost acts as a slider that enable to share the benefits of the relationship in any proportion between the two parties (cf. §2.1.2 on bargaining). Hence, when \( u \) is large, it is the agent who has most bargaining power.

To convince the agent to sign the contract in the first place, the principal must offer him an expected utility at least as great as his opportunity cost \( u \). This *individual rationality* (IR) condition is in fact a participation constraint:

\[
u(w) - c(e) \geq u \iff w \geq w(e) \equiv u^{-1}(u + c(e))
\]
Since \( u \) and \( c \) are increasing, so is \( w \), meaning that the higher the effort one wishes to implement, the higher must be the compensation.

**First Best**

The principal can now maximize her objective \( \pi = q(e) - w \) over contracts \((e, w)\) satisfying the participation constraint (IR). Since she likes money too, it is optimal for her to saturate the participation constraint (2.1) by restricting attention to contracts \((e, \underline{w}(e))\) i.e., pay the minimum acceptable wage. The principal’s program thus becomes

\[
\max_e q(e) - \underline{w}(e).
\]

(2.2)

The FOC for (2.2) is

\[
q' = \underline{w}' = \frac{c'}{u'} \iff c' = q'u'.
\]

(2.3)

i.e., the marginal utility of taking one minute of rest \( c' \) equals the marginal value of money \( u' \) times the additional production \( q' \) of one additional minute of effort. Since \( q \) and \( u \) are both concave, the RHS of (2.3) is decreasing while \( c \) being convex implies that the LHS is increasing, there is thus a unique solution \( e^* \) called the first-best effort. The first-best contract is \((e^*, \underline{w}(e^*))\) and yields the final profit \( \pi^* \equiv q(e^*) - \underline{w}(e^*) \).

**2.1.2 Moral Hazard**

Most often, effort is too complex to be monitored closely\(^2\) but there does exist a minimal effort \( e \) that can be required from an employee i.e., failure to perform at this level is a juridically acceptable reason for firing him without salary.\(^3\) In such a situation, the wage scheme becomes flat i.e., the agent is paid a salary \( w \). Since his utility is \( u(w) - c(e) \), he has no incentive to work harder than the contractual minimum, thus he will perform \( e \). The minimal wage satisfying the participation constraint is then \( \underline{w}(e) \) and the principal earns \( \pi \equiv q(e) - \underline{w}(e) \). The literature

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\(^2\)What is meant here is that although it would be technically possible to perform such a monitoring, it would be so costly that it is better not undertaken.

\(^3\)Notice that minimal effort can take several meanings. When the job is painful it is presence at the work place that will determine this minimum level (absence is easily verified). If the agent loves his work, the minimum level is in fact the threshold above which he refuses to sacrifice family life without additional compensation.
often assumes $e = 0$ to simplify matters but it is important to remember that the principal has also an opportunity cost of entering into the relationship with the agent which plays a symmetrical role to $\underline{u}$.

The very idea that moral hazard is an issue translates into $e < e^*$, the fact that the principal would the agent to perform more than the minimally contractible effort. This also implies that $\underline{\pi} < \pi^*$ (recall that the profit $q(e) - \underline{w}(e)$ is concave with a maximum at $e^*$).

**Verifiable Output**

Although effort is not verifiable, it can be the case that the output produced can be contracted upon. Since there is a one-to-one relationship between effort and production, any effort $\hat{e}$ is uniquely associated to the output $\hat{q} \equiv q(\hat{e})$ and to the compensation $\hat{w} \equiv \underline{w}(\hat{e})$. Let us then consider the following contract “produce at least $\hat{q}$ and i’ll pay you $\hat{w} + 1\notin$, otherwise nothing”. It is readily observed that upon accepting this offer, the agent chooses to perform exactly $\hat{e}$. As he obtains a utility level $u(\hat{w}(\hat{e}) + 1) - c(\hat{e}) > u(\underline{w}(\hat{e})) - c(\hat{e}) = u$, he will accept the contract in the first place. Hence the principal can finely tune the contract (reduce the $1\notin$ bonus) to leave no more than $u$ to the agent. Choosing $\hat{e} = e^*$ implies that the first-best can be achieved although effort is not contractible.

The previous scheme is however quite sensitive to the precise observation of $q(\hat{e})$ and would fail to work properly if risk was taken into account. A standard *piece rate* scheme can achieve the same outcome; it consist of a base salary $\hat{w}$ and a piece rate factor $\beta$ so that $w(q) = \hat{w} + \beta q$. Upon signing such a contract, the agent has utility $u(\hat{w} + \beta q(e)) - c(e)$ and is thus lead to expand the first best effort $e^*$ whenever the principal sets $\beta = 1$ i.e., makes the agent the *residual claimant* of the activity. The base salary is then set so as to satisfy the participation constraint i.e., $u(\hat{w} + q(e^*)) = u + c(e^*) \Leftrightarrow \hat{w} = w(e^*) - q(e^*) = -\pi^*$.

**Franchising**

Up to a slight change in the sequentiality of payments, the previous scheme is identical to franchising or “selling the store”. Here, the agent pays the principal $\pi^*$ up-front (or its expectation if there is uncertainty) and then becomes *residual claimant* of the activity. Although effort may not be contractible, the agent's
wage is now $w(e) = -\pi^* + q(e)$ so that his utility is $u(q(e) - \pi^*) - c(e)$. The optimal effort solves the first-best FOC (2.3) and since $q(e) - \pi^* = w(e)$ at $e^*$, the solution is the first-best effort $e^*$. The obstacle to such a clever method is limited liability i.e., in most cases where moral hazard matters, the agent is poor compared to the principal and cannot pay upfront the potentially large amount $\pi^*$.

**Bargaining power**

It is realistic to assume that in their first encounter it is the principal who makes a “take-it-or-leave-it” offer to the agent and not the reverse. Once the agent has been working for her a while, he has acquired a valuable human capital for the firm and thus may be able to dictate his conditions for the next period.

In that case, the agent will not buy the store but offer the principal the contract $(e, w)$ maximizing his own utility $u(w) - c(e)$ under the constraint that the principal accepts the offer. Turning down the agent, the principal could do the job herself or hire a new agent but in any case she would miss the participation of the old and more experienced agent and she would end up earning a lower profit $\underline{\pi} < \pi^*$; this level $\underline{\pi}$ therefore represents her opportunity cost. The offer of the agent must satisfy $q(e) - w \geq \underline{\pi}$ for it to be accepted by the principal (participation constraint). Like the principal, he likes money, thus proposes the minimally acceptable offer, that saturating her participation constraint.

The agent’s utility is thus $u(q(e) - \pi) - c(e)$ and his optimal effort is again efficient.\(^4\) Under this scheme, the agent reaches a utility level of $u^* > u$ since all gains of the economic activity are passed from the principal to him (by duality $\pi < \pi^* \Leftrightarrow u < u^*$). Hence, by sliding the reservation utility from $u$ to $u^*$, we are able to implement any surplus distribution between the principal and the agent (cf. §?? on bargaining). The “take-it-or-leave-it” hypothesis is thus best seen as a tool to simplify the analysis without restricting its generality.

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\(^4\)Although the equation to solve is still $q'u' = c'$, it is evaluated at a different income level; hence wealth effects could alter the first best effort but it remains first-best conditional on $\pi$, just like $e^*$ was efficient conditional on $u$.\[^26\]
Risk and Uncertainty

In most basic moral hazard settings, the agent applies effort on some project or activity which later on yields a result or profit. It is clear that many unpredictable events like the weather or macro-economic shocks interfere with the work of the agent to increase or decrease the magnitude of his production, to change a winning project into a losing one. Hence the one-to-one relationship between effort and output is lost. This means that incentives based on output force the agent to bear risk: although he worked hard, the resulting performance might be adverse so that pay might be lower than expected. Whenever the agent is risk-averse, the use of incentives becomes more costly to the principal because she must compensate the agent for his risk bearing.

2.2 Managerial Incentives

Before presenting the general model of moral hazard, we develop here a simple model that captures the issues of risk and incentives. We show how contractual terms are optimally distorted towards lower effort to provide the agent with some insurance. The resulting optimal contract is called second-best because the associated profit falls short of the first-best level, the difference is called the agency cost of moral hazard. We then turn to a series of extensions that account for realistic features absent from the base model.

We first derive the optimal incentive scheme as a decreasing function of the uncertainty surrounding the agent and his degree of risk aversion. Next, we show that if the observable performance measure on which incentives can be build is loosely aligned with the principal’s real objective then incentives are watered down to impeach the agent to “game” the scheme. Next we analyze succinctly relative performance measure and show that evaluation against peers, although fundamentally a worse instrument than piece rates, does eliminate firm specific risks unlike absolute performance measures; for that reason, “employee of the month” and promotions can become optimal incentive schemes. The next step is job design and whether people ought to work alone or in team, so to say. We show that team work dominates individual accountability when uncertainty is not so much of an issue and technological complexity makes it difficult to trace
specific effort from outcomes. Lastly, we deal with implicit incentives or career concerns, the fact that one works harder at the beginning of his professional life to signal high ability and enjoy later on a better position or earnings.

2.2.1 Individual Compensation

To simplify our study of uncertainty we make a number of simplifying assumptions in this section. The equivalent monetary cost of effort is quadratic and after normalizing adequately the measure of a unitary effort, becomes \( c(e) = \frac{1}{2} e^2 \). The manager has a zero opportunity cost \((u = 0)\) and constant risk aversion \( \rho \); as we saw in §1.1.3 (eq. 1.2), if he earns a random normally distributed income \( \tilde{x} \), he maximizes \( u \equiv E[\tilde{x}] - \frac{\rho}{2} \sqrt{\tilde{x}} \). We posit a constant productivity \( \gamma \) of labour and an additive noise, blurring the relation between input and output i.e., \( q = \gamma e + \tilde{e} \). We further assume that the noise \( \tilde{e} \) follows a centered normal law with variance \( \sigma^2 \). To conclude, the contractible performance measure is an imperfect signal of effort BUT, in this section, the risk or randomness is independent of effort.

The wage scheme used by the owner consists of piece rate or bonus factor \( \beta \) and a base salary \( w \), so that total (random) wage is \( \tilde{w} = w + \beta (\gamma e + \tilde{e}) \) leading to an expected utility of\(^2\)

\[
 u(e) = E[\tilde{w}] - \frac{1}{2} \rho \sqrt{\tilde{w}} - \frac{1}{2} e^2 = w + \beta \gamma e - \frac{1}{2} e^2 - \frac{1}{2} \rho \sigma^2 \beta^2 \gamma^2
\]  

(2.4)

Observe that a greater bonus factor \( \beta \) increases the manager’s risk exposure; the last term in (2.4) is the corresponding risk premium which is independent of effort due to the assumed additivity in the performance measure. The optimal level of activity maximizes \( u \) by equating marginal wage \( \beta \) to marginal cost of effort i.e., \( \hat{e}(\beta) = \beta \gamma \). As intuition would suggests, effort increases with the bonus factor and productivity. Plugging the optimal activity into (2.4), the agent’s expected utility simplifies into \( u(\hat{e}) = w - \frac{1}{2} \beta^2 \gamma^2 (\rho \sigma^2 - 1) \). The agent is willing to accept the wage scheme if this is positive thus the principal sets \( w(\beta) = \frac{1}{2} \beta^2 \gamma^2 (\rho \sigma^2 - 1) \) so as to saturates this participation constraint.

Summarizing, a bonus \( \beta \) motivates the manager to expand effort \( e = \beta \gamma \) at expected cost

\[
 \tilde{w}(e) = w(\beta) + \beta \gamma \hat{e}(\beta) = \frac{1}{2} (1 + \rho \sigma^2) \beta^2 \gamma^2 = \frac{1}{2} (1 + \rho \sigma^2) e^2
\]  

(2.5)
We can now solve for the optimal contract of the firm. Since expected output $Q = \mathbb{E}[q]$ is linear in effort with $Q = \gamma e$, the cost function is $C(Q) = \frac{1+\rho \sigma^2}{2 \gamma^2} Q^2$. Profit $\pi = Q - C(Q)$ is thus maximized for $\hat{Q} = \frac{\gamma^2}{1+\rho \sigma^2}$, yielding a maximum $\hat{\pi} = \frac{\gamma^2}{2(1+\rho \sigma^2)}$.

To relate these findings to the first-best seen in the previous section, recall that the true cost of effort to the manager is $c(e) = \frac{1}{2} e^2 \leq \hat{w}(e)$. There is equality with (2.5) if either $\sigma = 0$ (no risk) or $\rho = 0$ (risk-neutrality). The objective at the first-best is $\pi^* = \gamma^2 / 2$ so that the agency cost of moral hazard, expressed in percentage, is $\frac{\pi^* - \pi}{\pi^*} = \frac{\rho \sigma^2}{1+\rho \sigma^2}$. Notice that the quantity objective might be replaced by a quality one or a reduction of marginal cost. In all cases, the presence of moral hazard generates an additional diseconomy of scale since the marginal cost of reaching such the objective is an increasing function of the objective level.

2.2.2 Misaligned Incentives

The ultimate objective of the firm is profit; yet no single employee has a definite weight over it so that the yearly profit is a poor signal of one person’s past effort and is therefore not a good instrument to use in order to motivate hard work. The numerous objectives that concur to profit are quality, sales, human capital acquisition or cooperativeness. Most are too complex to be contracted, so that realistic objective performance measure are only positively correlated with the firm’s objective for the particular employee. We shall see within the previous model that explicit incentives will be optimally low powered to avoid distorting the agent’s effort towards unnecessary goals. More generally, multiple instruments such as yearly cash bonus, promotion or job design enable to span better the fundamental objectives of the firm and avoid misaligned incentives.

Suppose that the valuable output is $q = e + \tilde{e}$ (assuming unitary productivity) but that the observable contractible performance measure is $x = \theta e + \tilde{e}$ where $\theta$ is an index of the divergence between the principal and the agent objectives. Prior to the relationship, this parameter is unknown to all; we assume variance $\sigma^2$ and a unitary mean so that, on average, the performance measure is well designed. Once the agent starts to work for the principal, he becomes knowledge-
able about the firm technology and thus learns the true realized value of \( \theta \) i.e., whether his effort tend to over or under emphasize the observable performance measure (with respect to its real impact on profits).

Based on our previous model, it is immediate to see that a bonus \( \beta \) based on \( x \) leads the informed employee to choose effort \( \hat{e} = \beta \theta \) and expect total utility \( u(\hat{e}) = \underline{w} - \frac{1}{2} \beta^2 (\rho \sigma^2 - \theta^2) \). The base salary guaranteeing ex-ante that the agent is willing to sign is thus \( \underline{w} = \mathbb{E} \left[ \frac{1}{2} \beta^2 (\rho \sigma^2 - \theta^2) \right] = \frac{1}{2} \beta^2 (\rho \sigma^2 - \sigma^2_\theta - 1) \) as \( \mathbb{E}[\theta^2] = \sigma^2_\theta + \mathbb{E}[\theta]^2 \). Since expected effort is \( \mathbb{E}[e] = \beta \), the expected cost for the principal is then an extension of (2.5):

\[
\hat{w}(e) = \underline{w} + \beta \mathbb{E}[x] = \underline{w} + \beta \mathbb{E}[\theta e] = \underline{w} + \beta^2 \mathbb{E}[\theta^2] = \frac{1}{2} (1 + \rho \sigma^2 + \sigma^2_\theta) e^2 \quad (2.6)
\]

Thus, even for a risk neutral agent \( \rho = 0 \), the principal’s adjusted cost of effort is increased by the agent’s ability to exploit his insider information at her expense (“gaming”). Whatever the value of the output, optimal incentives will be deliberately muted and optimal effort will be reduced further (as compared to the previous analysis).

2.2.3 Rank-Order Tournaments

Instead of relying on an absolute performance to reward an agent, one can use a relative performance by comparing an agent against a yardstick such as the previous year sales or against peers both within and outside the firm. Rank order refers to the fact that the margin of winning does not affect the level of compensation. There is thus a metering or recording advantage of relative performance from being an ordinal measure as opposed to piece rate which is a cardinal measure. Relative measures are also more flexible in the sense that if the environment changes e.g., technology improves, then all the peers are likely to be affected in the same manner so that incentives remain unaffected quite differently from the case of a piece rate scheme whose absolute bonus become automatically distorted.

We analyze the simple setting where two agents with current salary \( \underline{w} \) compete for a higher rank position paying a bonus \( \beta \). The first agent’s wage is then the random \( \hat{w}_1 = \underline{w} + \beta q_1 > q_2 \) which is a binomial variable. The probability of winning is \( \mathbb{E}[q_1 > q_2] = \mathbb{E}[\tilde{e}_1 - \tilde{e}_2 > e_2 - e_1] = H(e_1 - e_2) \) where \( H \) is the law of
a centered normal variable with variance $2\sigma^2$. The expected wage is thus $w + \beta H(e_1 - e_2)$ while its variance is $\beta^2 H(e_1 - e_2)(1 - F(e_1 - e_2))$ which, as a function of the bonus, is less risky than a piece rate scheme. As before, each agent has constant risk aversion $\rho$, thus maximizes

$$u(e_1) = w + \beta H(e_1 - e_2) - 2\rho \beta^2 H(e_1 - e_2)(1 - H(e_1 - e_2)) - \frac{1}{2} e_1^2$$

The FOC of optimal effort is $\beta f(e_1 - e_2) = e_1 + h(e_1 - e_2)(1 - 2H(e_1 - e_2))$. Since all the setting is symmetric, so is the equilibrium, hence the FOC reads $e_1^* = e_2^* = \beta h(0)$ as $H(0) = \frac{1}{2}$. Recalling that $H$ is the law of $\bar{\epsilon}_1 - \bar{\epsilon}_2$, one can compute $h(0) = \frac{1}{2\sigma \sqrt{\pi}}$. Hence, the greater the noisiness of the performance measure, the lower is the induced effort. To induce high effort, it is thus necessary to set a large bonus and force the agent to support a greater risk. As we shall now see, tournament ends up being more costly.

The salary is set to solve the agent participation constraint $u(e^*) = 0$ i.e., $w = -\frac{1}{2} \beta + \frac{1}{2} \rho \beta^2 + \frac{1}{2} (e^*)^2$. The total cost of achieving effort level $e$ per agent is thus $c(e) = w + \frac{1}{2} \beta = \frac{1}{2} \rho h(0)^2 + \frac{1}{2} e^2 = \frac{1}{2} (1 + 4\pi \rho \sigma^2) e^2$ as $\frac{1}{h(0)^2} = 4\pi \sigma^2$.

Evaluation against a yardstick $\bar{q}$ is a form of contest quite similar to the rank order tournament yielding a twice lower risk premium.\(^6\) Comparing the reward schemes is now easy in our setting since we only need looking at the risk premium factor in each cost formula. Since $4\pi \approx 12.6$, the risk premium is greatest under tournament, then twice lower under yardstick and lowest under piece rate. As soon as uncertainty matters, the tournament (relative evaluation) is the worse method of providing incentive, followed by yardstick while the direct evaluation enabled by piece rate is best; although all are inefficient since moral hazard cannot be eliminated as it must be traded against insurance.

However, if the noise blurring the observation of a worker’s effort is made of a firm specific component plus an idiosyncratic component i.e., $q_1 = e_1 + \bar{\eta} + \bar{\epsilon}_1$ then the variance factor in the piece rate cost is $\sigma = \sigma_\eta^2 + \sigma_\epsilon^2$ while there is no change in the case of a tournament because the firm specific components cancel out. It is now clear that the tournament is optimal whenever the firm specific

\(^6\)The case of yardstick evaluation is similar. One replaces $e_2$ by the fixed objective $\bar{q}$ to obtain the same FOC of effort. Setting $\bar{q} = \beta h(0)$ makes $e = \bar{q}$ the solution of the FOC. The only difference to derive the cost formula is that $1/h(0)^2$ is half the previous value because $H$ is the law of $\bar{\epsilon}_1$ only.
noise is greater than the idiosyncratic one i.e., $\sigma^2_\eta > (4\pi - 1)\sigma^2_\epsilon$. Notice that the yardstick evaluation is not immune against firm shocks. This results rationalizes the use of tournaments for managers because their effort relate to firm wide strategies who are particularly sensitive to external shocks. Workers at lower levels of the hierarchy operate in an environment relatively safe from external influences so that piece rate schemes are optimal for them. Lastly, our previous observation that tournaments generate more income variance than piece rate schemes ($\pi \sigma^2 e^2$ vs. $\sigma^2 e^2$), means that “risk averters” prefer piece rate schemes while “risk-lovers” prefer tournaments. This rationalizes the fact that “risk averters” occupy lower levels of the hierarchy and are paid according to their productivity (with a piece rate scheme) while “risk-lovers” choose risky occupations in which few win very large prizes.

### 2.2.4 Multi-Tasking

Corts (2007) studies team vs individual compensation when agents must perform several tasks (cf. §??). We consider two functions that apply to two product lines $\alpha$ and $\beta$; together they generate four activities. At a given level of employment, say two managers, an assignment scheme must be devised to perform all four activities. Assuming high returns from specialization, each of the two agents performs two tasks.

The effort level in any activity has a cost $c(e) = \frac{1}{2} e^2$ and yield a one-to-one monetary return for the owner i.e., $\pi = e_1 + e_2 + e_3 + e_4$. The performance measure of the two products are $q_\alpha = \gamma_1 e_1 + \gamma_2 e_2 + \epsilon_\alpha$ and $q_\beta = \gamma_3 e_3 + \gamma_4 e_4 + \epsilon_\beta$ where $\epsilon_\alpha$ and $\epsilon_\beta$ are independent noises blurring the observations. We cannot normalize the productivities to unity as before unless they are identical, thus we shall later on assume $\gamma_1 = \gamma_3 = 1$ and $\gamma_2 = \gamma_4 = \gamma \geq 1$ where the latter parameter measures the severity of the multi-task problem. As usual, the wage scheme includes a base salary and bonuses related to the observable outputs i.e., $w = \underline{w} + \alpha q_\alpha + \beta q_\beta$. As before, the salary $\underline{w}$ will be tuned to meet the participation constraint. Assuming that the managers are risk averse with (identical) coefficient of absolute risk aversion $\rho$, their gross expected utility is $u(e) = \underline{w} - \frac{1}{2} \rho \sigma^2 (a^2 + b^2) + \alpha (\gamma_1 e_1 + \gamma_2 e_2) + \beta (\gamma_3 e_3 + \gamma_4 e_4)$ which is an immediate extension of (2.4) except for the omission of effort costs.
If an agent works in a team, he must provide the pair \((e_1, e_3)\) or \((e_2, e_4)\)\(^7\) while if he works alone on a product line, he must provide the pair \((e_1, e_2)\) or \((e_3, e_4)\). In team work, the optimal level of activity \(e_1\) maximizes \(\alpha \gamma_1 e_1 - \frac{1}{2} e_1^2\), thus he chooses \(e_1 = \alpha \gamma_1\) and by symmetry \(e_3 = \beta \gamma_3\). Efficiency commands a unit effort in each activity; this can be achieved by tuning the appropriate bonus factor for that agent, whatever the productivity \(\gamma\) may be in each activity. This is the advantage of team work as we shall see right-away. Since the owner must compensate the agent for the risk burden, her expected cost is \(\frac{1}{2} (1 + \rho \sigma^2 / \gamma_1^2) e_1^2\) as in (2.5) up to the productivity factor \(\gamma_1\). The profit over the first activity is thus

\[
\Pi_1 = e_1 - \frac{1}{2} (1 + \rho \sigma^2 / \gamma_1^2) e_1^2 = \alpha \gamma_1 - \frac{1}{2} (\rho \sigma^2 + \gamma_1^2) \alpha^2
\]

so that the optimal bonus factor is \(\alpha = \frac{\gamma_1}{\rho \sigma^2 + \gamma_1^2}\) and the maximum profit reduces to \(\Pi_1 = \frac{1}{2} \frac{\gamma_1^2}{\rho \sigma^2 + \gamma_1^2}\). By symmetry for the third activity, \(\Pi_3 = \frac{1}{2} \frac{\gamma_3^2}{\rho \sigma^2 + \gamma_3^2}\). The case of the other agent is identical. Using the normalization of the productivities and summing the four activities, we obtain \(\Pi_{\text{team}} = \frac{1}{\rho \sigma^2 + 1} + \frac{\gamma_1^2}{\rho \sigma^2 + \gamma_1^2}\).

Let us study now individual line work. The \(\alpha\) product manager chooses effort \(e_1\) to maximize \(\alpha \gamma_1 e_1 - \frac{1}{2} e_1^2\), hence \(e_1 = \alpha \gamma_1\). Observe now that his optimal choice for the other task is \(e_2 = \alpha \gamma_2\) because his pay depends only on the \(\alpha\) output.\(^8\) Incentives are thus worse aligned but there is also less risk exposure since \(\beta = 0\) for that the \(\alpha\) product manager. The profit over product \(\alpha\) is thus

\[
\Pi_\alpha = e_1 + e_2 - \frac{1}{2} (e_1^2 + e_2^2) - \frac{1}{2} \rho \sigma^2 \alpha^2 = \alpha (\gamma_1 + \gamma_2) - \frac{1}{2} (\rho \sigma^2 + \gamma_1^2 + \gamma_2^2) \alpha^2
\]

so that the optimal parameter is \(\alpha = \frac{\gamma_1 + \gamma_2}{\rho \sigma^2 + \gamma_1^2 + \gamma_2^2}\) and the maximum profit reduces to \(\Pi_\alpha = \frac{1}{2} \frac{(\gamma_1 + \gamma_2)^2}{\rho \sigma^2 + \gamma_1^2 + \gamma_2^2}\). Using the normalization of the productivities and the perfect symmetry for the other product, we obtain total profit \(\Pi_{\text{ind}} = \frac{(1 + \gamma)^2}{\rho \sigma^2 + 1 + \gamma^2}\). One can show that\(^9\) \(\Pi_{\text{team}} > \Pi_{\text{ind}}\) if \(\gamma > 3 + 4 \rho \sigma^2\). We can conclude:

\(^7\)W.l.o.g. that the same agent is in charge of the activity whose productivity is one while the other looks after the activity whose productivity is \(\gamma\). This can be checked from the final profit \(\Pi_{\text{team}}\) formula.

\(^8\)Indeed there is no point to relate his pay to the result of line \(\beta\) since this would only add a risk burden given that he does not work on that line.

\(^9\)W.l.o.g. \(\sigma = 1\), \(\Pi_{\text{ind}} - \Pi_{\text{team}}\) is proportional to \(2 \rho^2 - (\gamma - 1)^2 \gamma + 2 \rho (\gamma^2 + 1 - \gamma)\). The meaningful
When the degree of risk aversion is low, or when signals of effort are informative or when the multi-task problem is serious, team work dominates individual accountability.

### 2.2.5 Career Concerns

A manager can be motivated by explicit incentives such as performance based bonuses or promotions, but also by the implicit incentives channeled through the labor market. If innate ability, which is highly variable, was easily observed (could be proved), managers would always be paid according to their productivity. Education is one way to signal this personal characteristic because of the differential cost to acquire human capital (cf. §3.1.3 on signaling). However people change jobs several times (on average) during their first decade of professional life, thus the market valuation of their ability comes to dominate that incorporated in the diploma. The fact that the market is able to infer a good estimate of real productivity out of observable performances gives rise to a different kind of signaling: people work hard as juniors in order to achieve great performance, signal their worth and ultimately achieve better senior positions. The incentive at work here is implicit because the market does not conscientiously design the inference process revealing abilities and it is also dynamic as future pay is based on past performance. We follow here the seminal contribution of Holmstrom (1982). On top of the moral hazard seen before (effort is delegated), the fact that ability is a private information to the worker introduces an adverse selection dimension to the problem.

Abilities in the population are measured by a zero mean index so as to capture superior or inferior individual ability. The agent whose ability is $\theta$ expands effort $e$ in his current occupation. We keep referring to the market or his future employer as the principal. The observable performance measure is $q = \theta + e + \tilde{\epsilon}$ where $\tilde{\epsilon}$ is a white noise as before. From the point of view of the principal, the ability is a centered normal variable with variance $\sigma^2_{\theta}$. Let us denote $\tau \equiv \frac{\sigma^2_{\theta}}{\sigma^2 + \sigma^2_{\theta}} \leq \frac{3}{4}$. Solution to the quadratic equation in $\rho$ is is a convex increasing function $f(\gamma)$ satisfying $f(1) = 0$ and asymptotic from above to $\frac{\gamma - 3}{4}$. Hence $\Pi_{\text{team}} > \Pi_{\text{ind}} \Leftrightarrow \rho < f(\gamma)$ which we approximate in the text by the asymptote.
1, a (positive) measure of the informativeness of the signal $q$ upon the ability $\theta$. Upon observing an output $q$ and anticipating an effort level $\hat{e}$, the principal derives a realization of the random variable $\theta + \tilde{\epsilon}$, he thus infers a new estimate of the underlying ability $\mathbb{E} [\theta | q, \hat{e}] = \tau(q - \hat{e})$ computed using the statistical laws of the involved random variables.

Ex-ante, during the junior part of his working life, the agent with ability $\theta$ must choose how much effort to invest in the signaling activity; he knows that his ex-post wage, as a senior manager, will depend on the effort $\hat{e}$ anticipated by the principal and on the noise $\tilde{\epsilon}$ (through the performance $q$), his expected wage is thus $\mathbb{E}_e [w] = \mathbb{E}_e [\mathbb{E} [\theta | q, \hat{e}]] = \tau(\theta + e - \hat{e})$. Given the quadratic cost of effort $\frac{1}{2} e^2$, the optimal effort is $e^* = \tau$ which is increasing in the market’s ability to infer productivity out of the observable performance.\textsuperscript{10} At the limit where ability is fully observable i.e., $\tau = 1$, the effort is efficient since welfare is here $q - c(e)$.

Since ability is unknown to the market, there are returns to effort because performance influences the ability perception. The agent thus tries to bias the process of inference in his favor. However, in equilibrium the market anticipates the effort level and adjusts the output measure accordingly i.e., no one can fool the market. Yet, the agent is trapped in supplying the equilibrium level that is expected of him, because, as in a rat race, a lower supply of labour will bias the evaluation procedure against him. Including many periods of repeated effort and performance reveals that effort gradually decreases as the end of the professional life approaches because the market inference has been refined to the point of identifying exactly the ability i.e., there is nothing left to signal which means that effort becomes useless.

\textbf{2.3 State of Nature Approach}

In this section, the valuable output is $q = \Phi(e, \theta)$ where $\theta$ is a choice of “Nature” such as a macro-economic shock. In the case of automobile insurance, $\theta$ can take one of two values, “accident” or “none” and $e$ is the cautiousness taken by the driver (e.g., respect speed limits). The presence of uncertainty does not by itself ruin our hopes to get insurance. If the insurer could install a black box reg-

\textsuperscript{10}Absent from the model is the discount factor for future earning; clearly, the more impatient (eager to live by the day) is the employee, the lesser the implicit incentives.
istering every driving decisions, then effort (driving care) would be contractible because it could be verified after the occurrence of an accident; the first-best would be achieved. What is reality for airplanes and trucks is not yet implemented for individual cars so that moral hazard is an issue. Another basic example is when the agent is the manager of a firm. The parameter $\theta$ can then reflect the strength of demand for the good he is in charge of developing and producing and $e$ is the amount of time dedicated to this project.\textsuperscript{11} It is quite clear that the overall profit of the firm will depend on both the market demand and the care with which the agent managed the project; the interconnection between the two components is so complex that there is no way to decipher the whole sequence of actions that the agent took to judge whether he worked hard or not. This means that effort is unverifiable and since it cannot be inferred from the output, it becomes a free decision for the agent. To repeat ourselves, the asymmetry of information in settings of moral hazard forces the principal to delegate crucial decisions to the agent.

\section{2.3.1 The Second Best program}

As we just argued, in the majority of real cases it is too costly to perform an audit after production has occurred to discover how much the agent has worked (and pay him accordingly). Hence, the principal can only propose a wage contingent on the production level, knowing that the agent will choose its effort to maximize his expected utility (over the distribution of the state of nature). Our task in this section is to characterize the optimal such contract; Figure 2.1 is the game tree describing the relationship between the principal and the agent. The players are the principal $P$, the agent $A$ and a special player which is not strategic, nature $N$. The first player to move is the principal who offers a contract out of a large range of possibilities (represented by the cone); it is a rule $w(q)$ stipulating a wage for each level of output. Then the agent either refuses or accepts the contract and immediately after he chooses his effort (in our simple sketch he has only two choices, high and low). Later on Nature decides whether the state of the world is favorable or not which gives us the final output $q = \Phi(e, \theta)$ jointly determined by his effort and the state of nature nature.

\textsuperscript{11}The rest of the time can be dedicated to shirk or pursuing a personal objective i.e., different from that of the principal.
The Principal-Agent game of Figure 2.1 is solved by backward induction, a concept seen in §3. The agent’s ex-post utility (at stage 5) is $u(w(q)) - c(e)$ where $q = \Phi(e, \theta)$, hence the agent’s expected utility at the time where he has to choose his effort (stage 3) is $U(w, e) \equiv \mathbb{E}_\theta[u(w(\Phi(e, \theta)))] - c(e)$. Being rational, the agent will pick the effort $\hat{e}(w)$ maximizing $v(w, e)$ over the available choices. Moving one step backward (stage 2), the rational agent will accept the contract $w(q)$ if and only if it guarantees him more than its outside option i.e., $U(w, \hat{e}(w)) \geq u$. Provided that $w$ is attractive enough to be accepted by the agent, the principal’s ex-ante profit (at stage 1) is $\Pi(w) \equiv \mathbb{E}_\theta[\Phi(\hat{e}(w), \theta) - w(\Phi(\hat{e}(w), \theta))]$. This cumbersome formulation is presented as the following optimization problem:\footnote{We assume here that the principal is risk neutral i.e., cares for profit only; otherwise his objective would be the utility of ex-post profit.}

$$\max_{w(.)} \mathbb{E}_\theta[\Phi(\hat{e}, \theta) - w(\Phi(\hat{e}, \theta))] \text{ s.t. } \begin{cases} U(w, \hat{e}) \geq u \quad (IR) \\
\forall e \neq \hat{e}, U(w, e) \geq U(w, \hat{e}) \quad (IC) \end{cases}$$

where $(IC)$ is the incentive compatibility constraint stating that the agent always chooses the effort best for him given the contract $w(.)$ he signed and $(IR)$ is the individual rationality constraint that guarantees participation.

Since this program is very difficult to solve we study a simpler version and assume that the agent manages a risky project yielding either success (value 1€) or failure (value 0€). A contract is now a pair $(w_0, \beta)$ where $\beta$ is the bonus for
good results (e.g., stock options for managers). The effort positively influences the probability of success but with decreasing return to scale i.e., the probability \( q(e) \) is concave increasing. We assume separability of effort and income with \( u(e, w_0) = u(w_0) - c(e) \) where \( u \) is concave increasing (monetary risk aversion) and \( c \) is convex increasing.

### 2.3.2 Resolution

Let us first find out which contracts induce the agent to take some specific effort \( e \). The expected utility of the agent is 

\[
(1 - q(e))u(w_0) + q(e)u(w_0 + \beta) = u(w_0) + q(e)\frac{c'(e)}{q'(e)}
\]

hence the optimal effort solves 

\[
\Rightarrow w_0 = \hat{w}(e) = u^{-1}\left(u + c(e) - \frac{q(e)c'(e)}{q'(e)}\right) < w(e) = u^{-1}(u + c(e))
\]  

(2.8)

where \( \hat{w}' > 0 \) and \( \hat{w}'' > 0 \) (composition of two convex functions). The difference with \( w(.) \) characterizing the participation constraint without moral hazard (cf. eq. (2.1)) is the fraction term. The profit for the principal when implementing \( \hat{e} \) is then

\[
\Pi(e) = (1 - q(e))(0 - \hat{w}(e)) + q(e)(1 - \hat{w}(e) - \beta(\hat{w}(e), e))
\]

\[
= q(e) - \hat{w}(e) - q(e)\beta(\hat{w}(e), e)
\]  

(2.9)

In the first-best regime where effort is contractible, the profit with a contract \( (e, w(e)) \) is \( q(e) - w(e) \). The difference in the presence of moral hazard is twofold: one the one hand, there is the costlier bonus \( \beta(\hat{w}(e), e) \) that must be given to the agent to induce him to perform \( e \) while on the other hand, the base salary \( \hat{w} \) is cheaper. The last step for the resolution of (2.7) is to choose an optimal \( \hat{e} \) to maximize (2.9). The optimal effort solving program (2.7), known as the second best
effort, is smaller than the first-best one and we may conclude that the presence of moral hazard forces the principal to distort the optimal contract towards less effort since incentives are now more costly to provide.

If the agent is risk-neutral, the bonus implementing an effort $e$ is independent of the salary, it is $\beta(e) = \frac{c'(e)}{q(e)}$ while its associated salary is $\hat{w}(e) = \underline{u} + c(e) - \frac{q(e)c'(e)}{q'(e)}$. In that case, the principal’s payoff is $\Pi(e) = q(e)(1 - \beta) - \hat{w}(e) = q(e) - c(e) - \underline{u}$ which is the first best.

Pay for outputs, not inputs

Mirrlees (1974) shows the following paradox: if there existed an outcome $\hat{q}$ that would occur with zero probability after the first-best effort $e^*$ but would occur with positive probability for every lower effort then the principal would successfully offer the agent the following contract “I pay you $\underline{w}(e^*) + 1\epsilon$ but if $\hat{q}$ appears you will be executed”. Backward rationality tells us that $e^*$ is optimal under acceptance of the contract and that accepting is optimal in the first place. The purpose of this silly example is to show that rewards should not be linked to output but to input. Hence if a lazy effort increase the occurrence of some output then the contingent wage should be very low to deter laziness. Likewise if the optimal effort yield more often some output then the contingent wage should be very large to encourage this effort.

2.3.3 The Mirrlees Approach †

Holmstrom (1979) provides a powerful yet simple characterization of the optimal contract using the Mirrlees (1974) approach of turning random variables into distributions admitting densities. The production is a random variable $\tilde{q}$ whose law is $H(e, q) \equiv Pr(\tilde{q} \leq q \mid e)$ depends on the effort $e$ previously chosen by the agent. The density is $h(e, q) = \frac{\partial H(e, q)}{\partial q}$; we denote $h_e = \frac{\partial h}{\partial e}$.

From the agent’s ex-post utility $u(w) - c(e)$, we deduce the ex-ante utility conditional on the wage scheme $\omega$:

$$U(\omega, e) \equiv \int u(\omega(q)) h(e, q) \, dq - c(e)$$

Moral hazard means that the agents chooses effort $\hat{e}$ to maximize $U(\omega, e)$ i.e.,
satisfies the incentive compatibility constraint (IC)

\[ c'(e) = \int u(\omega(q)) h_e(e, q) \, dq \] (2.10)

Furthermore, he will accept the wage scheme \( \omega \) only if \( U(\omega, \hat{e}) \geq u \) (IR).

Likewise the ex-post utility of the principal being \( \pi(q - w) \), she expects \( \Pi(\omega, e) \equiv \int \pi(q - \omega(q)) h(e, q) \, dq \) when the agent has accepted the scheme \( \omega \). Her objective is thus to maximize \( \Pi(\omega, e) \) under the above (IC) and (IR) conditions. The Lagrangean (with non negative multipliers \( \lambda \) and \( \mu \)) is

\[ \mathcal{L} = \int \pi(q - \omega(q)) h(\hat{e}, q) \, dq + \lambda (U(\omega, \hat{e}) - u) + \mu \left( c'(\hat{e}) - \int u(\omega(q)) h_e(\hat{e}, q) \, dq \right) \]

and can be maximized point-wise in the variable \( w \) i.e., by solving \( \frac{\partial \mathcal{L}}{\partial w} = 0 \) to obtain

\[ -\pi'(q - w) + \lambda u'(w) h(e, q) + \mu u'(w) h_e(e, q) = 0 \Leftrightarrow \frac{\pi'(q - w)}{u'(w)} = \lambda + \frac{h_e(e, q)}{h(e, q)} \]

(2.11)

A first best situation, one without moral hazard, is found by maximizing \( \Pi(\omega, e) + \lambda U(\omega, e) \) in \( \omega \) and \( e \) for some multiplier \( \lambda \) (this is equivalent to maximize \( \Pi \) under the (IR) constraint). The optimal risk sharing is found by a point-wise maximization as above; the FOC for every \( q \) is

\[ \frac{\pi'(q - w)}{u'(w)} = \lambda \] (2.12)

and the solution is an increasing\(^{13}\) function \( \omega^*(q) \). The efficient effort \( e^* \) solves\(^{14}\)

\[ c'(e) = \int \left( u(\omega(q)) + \frac{\pi(q - \omega(q))}{\lambda} \right) h_e(e, q) \, dq \]

\[ \frac{\pi'(q - w)}{u'(w)} = \lambda \] (2.11)

---

\(^{13}\)Given that \( u \) and \( \pi \) are concave, \( \frac{\pi'(q - w)}{u'(w)} \) is increasing so that equation (2.12) has at most one solution in \( w \); as the LHS diminishes with \( q \) this solution increases with \( q \). We might however have \( \omega^*(q) = 0 \) over some interval of low \( q \)'s and \( \omega^*(q) = q \) over some interval of large \( q \)'s.

\(^{14}\)We can safely assume that the IR constraint is binding (\( \lambda > 0 \)) whenever \( c \) is convex, \( q \) is bounded and conceivable effort is unbounded, for under these hypothesis \( U(q, e) \) diverges to \(-\infty\) as effort increases.
To be able to compare the risk-sharing equations (2.11) and (2.12), Holmstrom (1979) first proves that \( \mu > 0 \); then using the fact that the RHS \( \frac{\pi'(q-w)}{u'(w)} \) is increasing in \( w \), he can deduce that \( \omega^*(q) \gtrless \hat{\omega}(q) \Leftrightarrow h_e \gtrless 0 \). The meaning of this result is that higher effort is promoted: indeed, at all productions whose probability is increased by effort \( (h_e > 0) \), the wage increase faster than the first best one while the reverse holds true at all productions whose probability is decreased by effort \( (h_e < 0) \). If we further assume that the ratio \( \frac{h_e}{h} \) is increasing in \( q \) then the second best \( \omega(q) \) is increasing.\(^{15}\) This motivation towards hard work is nevertheless countered by the need to share risk so that in the end the second best effort is inefficiently low. Indeed, comparing (2.10) and (2.13) reveals that \( \hat{e} < e^* \) if \( \int \pi(q - \omega(q)) h_e(e, q) \, dq > 0 \) which happens to be a consequence of \( \mu > 0 \).\(^{16}\)

### 2.3.4 Automobile Insurance

Since the second best program (2.7) is difficult to solve we present Rasmusen (2006)’s simple application to automobile theft insurance where the analytical solution can be derived. Assume that effort has two levels, care \( \bar{e} \) and no-care \( e \) generating theft probabilities of \( \frac{1}{2} \) and \( \frac{3}{4} \) respectively. The utility of the car is 12 and the owner is risk averse. An insurance contract \( C = (p, \delta) \) is a premium \( p \) and a reimbursement \( \delta \) of damages; the absence of contract is \( C_0 = (0, 0) \). In the absence of an insurance contract the expected utility under care is \( u(C_0, \bar{e}) = \frac{u(12)}{2} - c \) where \( c \) is the disutility of taking care (e.g., checking every night that doors are locked). Without care the utility is \( u(C_0, e) = \frac{u(12)}{4} \). Assuming \( c \) not too large, the non-insured car owner will optimally take care.

Observe that a contract can also be labeled \( C = (w_{NT}, w_T) \) where \( w_{NT} = 12 - p \) and \( w_T = \delta - p \) are the income level of the owner in the two possible states of nature (theft and no-theft). No insurance can thus be labeled \( C_0 = (12, 0) \). As

\(^{15}\)As in footnote 13, a higher \( q \) diminishes the LHS of (2.11) but raises the RHS so that the solution must increase. Notice that this added hypothesis is the MLRP seen in §1.3. Indeed, \( \frac{h_e}{h} = \frac{\partial \ln h}{\partial e} \) implies that for \( e < e' \) we have \( \ln \left( \frac{h(e, q)}{h(e', q)} \right) = \int_{e'}^{e} h_e (x, q) \, dx \), thus the MLRP is either that the LHS is increasing in \( q \) or that \( \frac{h_e}{h} \) is increasing in \( q \).

\(^{16}\)The equation determining \( \mu \) is \( \int \pi(q - \omega(q)) h_e(e, q) \, dq = -\mu \left\{ \int u(\omega(q)) h_{ee}(e, q) \, dq - c''(e) \right\} \) where the term in braces is the second order condition of utility maximization for the agent, which is necessarily non positive.
the probability of theft is 50% under care, the owner is indifferent among $C_0$ and $C_4 = (0, 12)$. On Figure 2.2 the indifference curve joining $C_0$ to $C_4$ passes below $C_1 = (6, 6)$ because the owner is risk-averse.

![Figure 2.2: Risk Sharing](image)

Competition among risk neutral insurers should drive them to offer the full insurance contract ($p = 6, \delta = 12$) denoted $C_1$ that leaves an insurer break even (provided that the policyholder takes care). The economic surplus $S = u(6) - \frac{u(12) - u(0)}{2}$ goes to the car owner.

Moral hazard appears immediately because the car owner has no more incentive to take care since $u(C_1, \bar{e}) = u(6) - c < u(C_1, e) = u(6)$: why take care if the car will be fully reimbursed whatever happens? Given this change of behavior, the insurer now expects losses of $6 - \frac{3}{4} \cdot 12 = -3$ and will raise the premium from 6 to 9 (contract $C_2$ on Figure 2.2). Under this contract the utility of the owner is $u(C_2, \bar{e}) = u(3)$. If this amount is less than $u(C_0, \bar{e})$ (as shown on Figure 2.2) then the owner won’t buy insurance.

Full insurance is thus incompatible with both the insurer IR constraint (offering a contract) and the owner IR constraint (agreeing to buy it). The optimal contract $C^* = (w^*_N, w^*_T)$ has to give an incentive to the owner to take care of its
car i.e., must satisfy

\[ \frac{u(w_{NT}) + u(w_T)}{2} - c \geq \frac{u(w_{NT}) + 3u(w_T)}{4} \]

\[ \Leftrightarrow 4c \leq u(w_{NT}) - u(w_T) = u(12 - p) - u(\delta - p) \]  

(IC)

i.e., the contract has to be below the bold dashed curve on Figure 2.2 known as the IC curve.

Since insurer competition bestow all the surplus on the owner, the equilibrium contract will leave the insurer break even i.e., be the intersection \( C_3 \) of the IC curve and the insurer IR line joining \( C_0 \) to \( C_4 \). Let us assume to simplify exposition that the optimal reimbursement is \( \delta^* = 10 \). Then the break even premium is \( p^* = 5 \) so that the income levels are \( w_{NT}^* = 7 \) and \( w_T^* = 5 \). The difference \( 12 - \delta^* = 2 \) is the deductible of the optimal insurance contract. The larger the deductible, the larger the risk supported by the policyholder thus the greater the care he takes of the insured item. Full insurance is rare because 12 stands for the value of finding, buying and bringing home the item while a traditional “full insurance” contract only repays the market value. Hence the owner has always a little incentive to take care of his property.

2.4 Renegotiation and Auditing

In this last section, we treat additional topics relating to the renegotiation of contracts and the game of auditing/cheating between taxpayers and the IRS.

Binary choice

Let us introduce the idea of renegotiation and its aftermath by way of a simple example. An individual earns \( w \) by working the regular time but can obtain a bonus \( \beta \) by working extra-hours although this has an opportunity cost \( c > \beta \) to him so that his current dominant choice is to perform no extra hours. Let us now introduce into the picture a car of objective (market) value \( p \) and subjective value \( \pi \) to the individual with \( p < \pi < \beta \) i.e., the car is more valuable than the market price \( p \) but lesser than the maximum salary. If \( \pi - p > c - \beta + w \) i.e., it is efficient for the individual to own the car, then a bank can lend \( p \) to the agent.
to buy the car with the promise to repay $p$ later on by working extra-hours. The reason why such a contract works is the following: by working hard, the agent makes enough money to repay the loan and enjoy the car, hence his final utility is $\pi - p + \beta - c > w$. If on the contrary he decides to shirk then he won't be able to repay the loan, the bank will seize the car and his final utility will be $w$. The previous condition is the incentive compatibility one.

If renegotiation is possible then when the bank faces a defaulter it cannot help but renegotiate. Indeed, the car being already used, it loses much of its market value thus it is better for the bank to accept the lower payment $w$ from a shirker (instead of $p$) than seizing and reselling for even less. This totally changes the incentives for work since the utility of leisure is now $w + \pi - w = \pi$ which can be greater than $\pi - p + \beta - c$. In that case, the agent optimally chooses to shirk and in response the bank ceases to offer a loan knowing that it will make a loss $p - w$. There is a market failure generated by the inevitability of renegotiation. Banks are therefore forced to be tough (commit to seize no matter what) in order to be able to operate a Pareto improving intermediation service.

Contractual renegotiation can be a problem because it is often unavoidable. If one party into an agency relation cannot engage into a long term relationship then contracts will be short-term and renewed frequently which is like the renegotiation of a long-term contract. This happens for regulators since the prices they impose on firms last no more than the regulatory period and there can be no insurance that the legislator or the government will not change its policy the next time. Likewise, a sovereign state, unless tied by a treaty like the EU one or by membership of the WTO, will easily renge a contract with a firm, whether national or foreign. Beyond these cases of forced renegotiation, we find the voluntary ones: if the parties agree to tear-up the contract and write a new one, no one can stop them.

**Continuous choice**

Let us use our formal set-up to analyze the consequences of renegotiation. Once the agent has exerted the second best effort $\hat{e}$, it not necessary anymore for the principal to use the costly bonus $\beta(\underline{w}(\hat{e}), \hat{e})$. Indeed the bonus was offered to motivate the agent although it was forcing him to support unwanted risk; now that effort has been expanded, the bonus has played its role and should be re-
moved before uncertainty resolves itself in order to fully insulate the agent from risk. Technically, the constant wage $\hat{w}$ solving $(1 - q(\hat{e})) u(w(\hat{e})) + q(\hat{e}) u(w(\hat{e}) + \beta) = u(\hat{w})$ is cheaper than the expected payment $w(\hat{e}) + q(\hat{e}) \beta$ stipulated in the original contract (this is so because $u$ is concave).

The principal just got trapped because the agent, anticipating this change to come, has no more incentive to produce the effort $\hat{e}$; he is actually fully conscious that his final payoff will be constant although it appears at first sight to contain a bonus. The rational agent therefore chooses the minimum level $\underline{e}$ instead of $\hat{e}$. This reasoning is true for any effort $e > \underline{e}$ because an incentive contract with a positive bonus $\beta(w_0, \hat{e})$ is always renegotiated to a nil one $\beta = 0$ which precludes (in equilibrium) the agent from performing $e$.

To resolve this time-inconsistency, the principal must offer a menu of contracts contingent on effort that make the agent indifferent between all efforts in $[\underline{e}; \hat{e}]$ and, given the equilibrium distribution of effort $\sigma$ over $[\underline{e}; \hat{e}]$, makes the principal indifferent between renegotiating and not. We shall not delve into the computations but it should be clear that the average effort implement under this scheme will fall below the second best one $\hat{e}$; thus we may say that committing to a single contract or committing to never renegotiate is helpful for the principal. In other words it pays “to be true to one’s word”.

Notice lastly, that if the agent leads the renegotiation\(^\text{17}\) and is rich enough to “buy the store” at price $\Pi(\hat{e})$, then renegotiation is not a problem anymore. Indeed the principal will accept a change in the contract only if the agent offers more than $\Pi(\hat{e})$ which can be achieved only by expanding the second best effort $\hat{e}$!

\textbf{Debt renegotiation}

We show here that the seller of an item has an incentive to finance her activity through debt to dilute the bargaining power of the buyer. We consider in turn the two cases where debt is not renegotiable and renegotiable to show that the latter fosters the strategic purpose of debt.

In most trading situation, the surplus generated by an exchange depends on many factors like future market demand or future price of oil not yet known

\(^\text{17}\)This reversal is not as extraordinary as it may seem: having provided effort the agent has become indispensable to the principal and might have acquire the bargaining power.
when the parties devise their trade. From an ex-ante point of view the surplus is a random variable \( x \in \mathbb{R}_+ \) whose distribution function is \( H(x) \) (\( h \) denotes the density). In this situation, the trading price has to be set through bargaining once the surplus is known to the parties. The seller and the maker share it according to their respective bargaining abilities \( \lambda \) and \( 1 - \lambda \).

Whenever the surplus \( x \) is inferior to the debt service \( F \), the seller is caught in the debt overhang problem thus she refuses to deal and closes her firm. As a consequence, the value of debt is only \((1 - H(F))F\). The value \( V \) of the firm for the seller is thus the value of her share \( 1 - \lambda \) of profits \( x - F \) when she does not go bankrupt plus the funds raised from the debt emission i.e.,

\[
V = \int_{F}^{\infty} (1 - \lambda)(x - F) \, dH(x) + (1 - H(F))F = (1 - \lambda) \int_{F}^{\infty} x \, dH(x) + \lambda(1 - H(F))F
\]  

The optimal level of debt \( F^* \) solves

\[
- (1 - \lambda) h(F) F + \lambda(1 - H(F)) - \lambda h(F) F = 0 \Leftrightarrow \lambda = \frac{h(F)F}{1 - H(F)}
\]

and is positive since the RHS is zero for \( F = 0 \).

If debt has been issued by a single bank then the event of bankruptcy is not so sure because the bank might prefer to renegotiate the debt service \( F \) down to \( x \). Credibly threatening not to trade, the seller gets all the surplus and use it integrally to repay the bank. The value of debt is now \( V_D = \int_{F}^{\infty} F \, dH(x) + \int_{0}^{F} x \, dH(x) \) so that the value of the firm for the maker becomes

\[
\tilde{V} = (1 - \lambda) \int_{F}^{\infty} (x - F) \, dH(x) + V_D = (1 - \lambda)E[x] + \lambda V_D
\]  

which is increasing function of \( F \), hence the optimal level of debt is maximum. Renegotiation fosters the strategic purpose of debt which is to reduce the bargaining power of the buyer. This is so because the buyer loses all of his bargaining power when the firm is in danger of going bankrupt.

### Audit

In all countries the tax authority (Internal Revenue Service in the US), is faced with “suspect” income declarations by tax payers. We can view the relation of a liable citizen (agent \( A \)) with the tax authority (principal \( P \)) as a game. Two forms can be thought of:

- **Simultaneity**: the tax authority chooses to audit or trust a declaration while
the citizen chooses to cheat or reveal its true income.

In the first game, the strategy of the tax authority is the probability of auditing $\sigma_P$ while the strategy of the citizen is the probability of cheating $\sigma_A$. Letting $t$ denote the income tax, $f$ the fine paid if caught cheating and $c$ the unit cost of audit, payoffs are

\[
\Pi_A(\sigma_P, \sigma_A) = -\sigma_A\sigma_P(t + f) - (1 - \sigma_A)t \tag{2.16}
\]

\[
\Pi_P(\sigma_P, \sigma_A) = \sigma_P(T - c + \sigma_Af) + (1 - \sigma_P)(1 - \sigma_A)t \tag{2.17}
\]

There is no pure strategy equilibrium since $\sigma_A = 1$ (cheat) triggers $\sigma_P = 1$ (audit) itself triggering $\sigma_A = 0$ (reveal). In that case there is no point to audit ($\sigma_P = 0$) and therefore the citizen is better off cheating ($\sigma_A = 1$). To solve this conundrum, we have to look for a mixed strategy equilibrium. The audit frequency $\sigma_P^*$ making the citizen indifferent between revealing and cheating solves

\[
\frac{\partial \Pi_A(\sigma_P, \sigma_A)}{\partial \sigma_A} = -\sigma_P(t + f) + t = 0 \quad \Rightarrow \quad \sigma_P^* = \frac{t}{t + f}
\]

and likewise

\[
\frac{\partial \Pi_P(\sigma_P, \sigma_A)}{\partial \sigma_P} = (t - c + \sigma_Af) - (1 - \sigma_A)t = 0 \quad \Rightarrow \quad \sigma_A^* = \frac{c}{t + f}
\]

is the cheating frequency making the tax authority indifferent between auditing and trusting. The final payoffs are $\Pi_A^* = -t$ and $\Pi_P^* = t - \frac{tc}{t + f}$ as if the tax authority was always auditing and citizens would therefore always pay their taxes. Still in equilibrium some people cheat and some do not while some are audited and some are not.

In the second game we are in fact assuming that the tax authority can credibly announce that it will audit $\alpha\%$ of all income declarations. Choosing $\alpha = \frac{t}{t + f} + 1\%$ forces each citizen to reveal because the probability of audit is now strictly superior to $\sigma_P^*$. Final payoffs are identical but no cheating takes place anymore thus the following year the tax authority will tend to (secretly) decrease $\alpha$ since
it is a costly action but then citizens will anticipate this and start to cheat again!!
Chapter 3

Adverse Selection

The private information held by an agent is crucial to determine the efficient decision in the agency relation. To make sure that this optimal action will be carried out, the principal must elicit this information. As intuition suggests, this can only be achieved by giving up an *information rent* to the agent (with respect to the complete information case). However this mechanically increases the marginal cost of production and creates a distortion or welfare loss. The principal must therefore arbitrate between productive efficiency and information acquisition.

This chapter contains two section. We first model the unraveling and signaling of information in a market and then study various models of screening where firms design multiple contracts that are offered to all their potential clients to take advantage of the induced self-selection.

3.1 Information Unraveling

In this section, the item for sale has a quality only known to its producer (service) or current owner (good). In Akerlof (1970)’s model of partial equilibrium for second-hand goods, a market failure occurs that can even lead to complete market breakdown. When there is a single seller, Levin and Tadelis (2005) show how the firm adapts its internal organization to resolve the information revelation conundrum. In an application to the theory of the firm, it is shown that the partnership structure can dominate the corporation structure if the quality of the service for sale is difficult to observe.

In this section we shall get a better understanding of the law on currencies
stating that “bad money drives out good”¹ or Groucho Marx joking “I wouldn’t want to belong to any club that is willing to give me membership” or businessmen saying that “trade is difficult in developing countries”.

3.1.1 Market for Used Cars

Akerlof (1970) observes that in many markets, buyers use statistics to judge the quality of prospective purchases. In that case, there is an incentive for sellers to offer poor quality merchandise; indeed the returns for selling good quality items accrue to the entire group whose statistic is affected rather than to the individual seller. As a result of this free riding, there tends to be a reduction in the average quality of goods and also in the size of the market.

A startling example is the large price difference between new cars and those which have just left the showroom. Automotive companies try their best to build perfectly good cars but there are always some copies that turn to be “lemons” (a colloquialism for defective used cars). Since the car is brand new, neither the buyer nor the dealer knows the exact quality of the car; there is symmetric information (or more precisely lack of). Things are different when you look at a used car because the current owner has probably learned whether her car was good or bad, she has acquired superior information by experiencing the car. There is now an asymmetry of information between buyers and sellers. Nevertheless, good cars and bad cars must still sell at the same price since it is impossible for a buyer to tell the difference. Rationally anticipating this fact, buyers will not accept a high price so that owners of good cars are trapped, they cannot sell their car for their real value nor for the price of a new car. Hence they remove their cars from the market so that actual sales are mostly lemons and since some potential sellers have withdrawn, the total number of transactions tend to be low.

Common Values

We will show in a simple setting how an extreme market failure may occur: if sellers and buyers value cars identically then the market completely breaks down.

¹If two coins have the same face value but are made from metals of unequal value, the cheaper will tend to drive the other out of circulation; the more valuable coin will be hoarded or used for foreign exchange instead of for domestic transactions.
To see this, consider a number of used cars whose quality or value \( \theta \) varies in the interval \([\bar{\theta}; \tilde{\theta}]\). If the price is \( \tilde{\theta} \), then all owners want to sell so that \( S(\tilde{\theta}) = 1 \), the total mass of cars while if the price is \( \bar{\theta} \), none of them find it profitable to sell, hence \( S(\bar{\theta}) = 0 \). By the same token, demand is maximum for \( p = \bar{\theta} \) and nil for \( p = \tilde{\theta} \). If, demand and supply are price responsive in the intuitive way then the two curves must intersect at some price \( p_1 \in ]\bar{\theta}; \tilde{\theta}[ \) for some quantity \( q_1 \) as shown by the plain lines on Figure 3.1. We shall see that this price cannot be an equilibrium price at which trade can take place.

![Figure 3.1: Market for Lemons](image)

When all owners offer their car for sale at price \( \tilde{\theta} \), the average quality of cars for sale is \( \theta_1 = \mathbb{E}[\theta] \), the average quality of all cars. When the price goes down to \( p_1 \), the supply shrinks to \( q_1 = S(p_1) < S(\tilde{\theta}) = 1 \) and we know for sure that some of the best cars have been pulled out of the market because their owners are the first to be hurt by the price decrease. Since potential buyers known that sellers are rational, they understand what’s going on and therefore revise their estimate of the average quality of cars for sale down to \( \theta_2 = \mathbb{E}[\theta | \theta \leq \theta_1] < \theta_1 \). As the demand for cars depends also on their quality, the effective demand curve will drop to reflect this updating. It is now clear that the price \( p_1 \) generates an excess supply since \( D(\theta_2, p_1) < D(\theta_1, p_1) = q_1 = S(p_1) \); it cannot be an equilibrium price. A further price drop down to \( p_2 \) will not solve the problem either. Indeed, it generates a lower supply, thus another downward revision of expected quality to some...
\[ \theta_3 = \mathbb{E}[q \mid \theta \leq \theta_2] < \theta_2 \] by buyers so that supply \( S(p_2) \) will still exceed the updated demand \( D(\theta_3, p_2) \).

This spiral of descending prices ends at \( \theta \) where cars of exactly this value (the worst of the market) are sold for that price. If instead of cars we consider the metallic content of coins we obtain the law on currencies. Likewise, Groucho Marx considering himself a man of the street realizes that his joining a posh club would lower the average status of the club to the point where it is not worthwhile anymore to join. As regards general business, the adverse selection problem just mentioned can be circumvented by certification or guarantees. Private intermediaries like Veritas or TUV Rheinland and public ones likes the national or international certifications authorities certify conformity with security, health rules or ISO standards of products in order to remove part of the uncertainty regarding their quality (cf. §??).\(^2\) Likewise, guarantees assure buyers that they will not end up with a “lemon”. Since fulfilling the guarantee is costly for the seller when he makes products of poor quality, offering a guarantee is also a signal of quality (cf. §??) for consumers and competitors. For instance, car dealers often propose used cars with a 6 month guarantee (at a higher price) after revising them extensively.

### Differing Values

Although shocking, the “lemons” example does not display a true market failure (there is no efficiency loss) since preferences were assumed identical between buyers and sellers so that ownership did not matter. Assume now that buyers value a car of quality \( \theta \) at \((1 + \lambda)\theta \) and are more numerous than sellers so that the latter capture all gains from trade. The market is in equilibrium at the price \( p^* \) if the expected value of car sold for that price is exactly that price i.e., if 
\[ (1 + \lambda)\mathbb{E}[\theta \mid \theta \leq p^*] = p^*. \]

Using the uniform distribution over \([\theta; \bar{\theta}]\), we obtain 
\[ p^* = \frac{1 + \lambda}{1 - \lambda} \theta. \]

The equilibrium is said to be partially pooling because all owners of type \( \theta \leq p^* \) act in the same way, they sell at price \( p^* \). Likewise, all owners of type \( \theta > p^* \) act identically by keeping their cars out of the market. Although the market

\(^2\)Certification requires a State (justice and police) strong enough to deter impersonators from defrauding clients and the very certifiers from either abusing their dominant position or selling false certificates. Examples still exists with the auditors in the Enron case or the credit rating agencies in the subprime meltdown.
is active, its outcome is inefficient as buyers value cars more than sellers; in a world of perfect information, all cars would be sold. This will occur here if \( p^* = \bar{\theta} \Leftrightarrow \lambda = \frac{\bar{\theta} - \theta}{\bar{\theta} + \theta} \), that is to say if the differential in intrinsic value between buyers and sellers is large enough to overcome the information asymmetry.

### 3.1.2 Corporation vs Partnership

In service activities requiring a large human capital such as lawyers, doctors, architects or auditors, firms are rather organized as partnership instead of corporations. Levin and Tadelis (2005) explain this choice as a consequence of the large information asymmetry present in these services, namely that the market (potential clients) observes imperfectly the quality.

In the activity under study, people’s intrinsic quality \( \theta \) is drawn randomly in \([0; 1]\). The market wage for that activity is an average \( w \in ]0; 1[ \) of quality over people active in the entire economy. Due to state regulation, workers must study to earn degrees and pass many exams that enable a potential employer to screen them and thus hire only the most talented people. If the ability of the last person hired is \( q \) then the firm size is \( 1 - q \) and its average quality is \( \mathbb{E} \left[ \theta \mid \theta \geq q \right] = \frac{1 + q}{2} \). Setting up a firm involves a fixed cost \( F \).

A priori, clients pay for the average quality of the firm but since it is imperfectly observed, there is a probability \( \lambda \) that they pay for an average expect quality \( q^e \). The expected price is thus

\[
p = (1 - \lambda)\mathbb{E} \left[ \theta \mid \theta \geq q \right] + \lambda q^e = (1 - \lambda) \frac{1 + q}{2} + \lambda q^e \tag{3.1}
\]

Parameter \( \lambda \) measures the extend of information asymmetry between sellers and buyers. By choosing its size (how many employees), a firm is choosing an average quality which affects the price more or less according to the severity of information asymmetries.

We can now study the difference between corporations and partnerships. A corporation \((A)\) maximizes the economic profit where employees cost is evaluated at the market wage, thus \( \pi = (1 - q)(p - w) - F \). The FOC for optimal size is

\[
0 = \frac{\partial \pi}{\partial q} = \frac{(1 - \lambda)(1 - q)}{2} - (p - w) = w - (1 - \lambda)q + \lambda q^e \tag{3.2}
\]
i.e., the expected ability of newest employee as assessed by the market is the market wage. As \( \frac{\partial \pi}{\partial q} \) is decreasing with \( q \), there is a unique solution to (3.2). In equilibrium, the market expectation for average quality is correct, thus \( q^e = \frac{1 + q}{2} \) so that

\[
w = (1 - \lambda)q + \lambda \frac{1 + q}{2} \Leftrightarrow q_A = \frac{2w - \lambda}{2 - \lambda} \leq w \tag{3.3}
\]

since \( w < 1 \) but with equality for \( \lambda = 0 \). Using the fact that in equilibrium \( p - w = \frac{(1-\lambda)(1-q)}{2} \), the equilibrium profit reads

\[
\pi_A = \frac{2(1-\lambda)(1-w)^2}{(2-\lambda)^2} - F \tag{3.4}
\]

A partnership \((B)\) maximizes its per-capita economic profit \( u = \frac{\pi}{1-q} = p - w - \frac{F}{1-q} \). Since \( \frac{\partial u}{\partial q} \propto (1-q) \frac{\partial \pi}{\partial q} + \pi \), the FOC for optimal size is \( \frac{\partial \pi}{\partial q} = \frac{-\pi}{1-q} = -u < 0 \). Since \( \frac{\partial \pi}{\partial q} \) is decreasing, the marginal ability of a partnership is larger than that of a corporation (solution of (3.2)); hence a partnership always employs less people than a corporation. Using (3.2) and taking out the market wage on both sides, the FOC for an optimal partnership reads \((1-\lambda)\theta + \lambda q^e = p - \frac{F}{1-q} \) i.e., the expected ability of the newest employee as assessed by the market is equal to the average gross profit share. In equilibrium, the market expectation is correct, thus \( q^e = \frac{1+q}{2} = p \) (from (3.1)) so that

\[
(1 - \lambda)q + \lambda \frac{1+q}{2} = \frac{1 + q}{2} - \frac{F}{1-q} \Leftrightarrow \frac{(1-\lambda)(1-q)}{2} = \frac{F}{1-q} \Leftrightarrow q_B = 1 - \sqrt{\frac{2F}{1-\lambda}} \tag{3.5}
\]

The partnership profit then simplifies to

\[
\pi_B = (1 - w)\sqrt{\frac{2F}{1-\lambda}} - \frac{2-\lambda}{1-\lambda}F \tag{3.6}
\]

Notice that our claim \( q_B > q_A \) is strictly equivalent to \( \pi_A > 0 \). Inspection of (3.4) reveals that if \( c \) or \( \lambda \) is too large, then the corporation is not sustainable and neither is a partnership, a result reminiscent of the lemons problem.

To compare the two organizational structures notice that for \( \lambda = 0 \), the corporation is efficient because it maximizes the true economic value of the firm i.e., \( q_A = w \). The resulting profit \( \pi_0 \equiv \frac{(1-w)^2}{2} - F \) is thus the overall maximum and we may conclude that corporation dominates partnership. Now, for \( \lambda_0 \equiv \frac{\pi_0}{F + \pi_0} \), we have \( \frac{2F}{1-\lambda_0} = 2(F + \pi_0) = (1-w)^2 \), hence \( q_B = w \) by (3.5). This time, it is the
partnership that maximizes the true economic value of the firm and is therefore the most efficient organization. By the implicit function theorem, there must exists a threshold $\lambda_1 \in ]0; \lambda_0[ \text{ such that } \pi_A = \pi_B$.\(^3\)

We conclude that the optimal organizational form is

- **Corporation** for low information asymmetries ($\lambda < \lambda_1$).
- **Partnership** for intermediate information asymmetries ($\lambda_1 \leq \lambda \leq \lambda_0$).
- **Neither** for large information asymmetries ($\lambda > \lambda_0$).

### 3.1.3 Signaling

As in the previous analysis, when an economically relevant personal characteristic is unknown to the market or other trade counterparts, he may want to act so as to *signal* his peculiarity. The concept is introduced by Spence (1973) in the context of education where a worker decides to acquire a costly education to signal his innate high productivity to potential employers.\(^4\) In the same vein, advertising, on top of being informative (cf. §??), can also be a signal of high quality. The idea applies also to a central bank whose monetary policy today is a signal of its willingness to accept inflation in the future. Employers then act according to the belief they form about future inflation.

The difficulty with signaling is that the valuable agent faces the threat that a cheap imitator might mimic his attitude and ruin his reputation— and the wage that goes with it—when results reveal the true characteristics. The only way to reveal credibly his true identity is to undertake something so costly that imitators would not dare follow the same path. Yet this commitment or bonding being costly it must be wisely chosen.

\[^3\]Letting $\bar{c} \equiv \frac{2(1-\lambda)(1-w)^2}{(2-\lambda)^2}$, it is possible to check that the solutions to equation $\pi_A = \pi_B$ are $c = \bar{c}$ where both profits are nil (which defines $\lambda_0$ implicitly) and $c = (1-\lambda)^2 \bar{c}$ which defines $\lambda_1$ implicitly.

\[^4\]This idea is independent but reinforces the traditional motive of education which is to increase one’s own ability.
Education

It is agreed that at least in the hard sciences, education adds cognitive skills to students. Their market productivity is thus increased and so is the market value of their labor. But education is also a process of socialization where one acquires skills such as the carrying out of assigned tasks, getting along with others, regularity, punctuality. Lastly, and this the point we pursue here, education is a filter in a world of imperfect information. Employers have a very poor idea of a candidate’s productivity although they may know the distribution. By delivering verifiable certificates, schools sort out candidates in small groups whose productivity is easier to assess. The education system thus reveals information about those who go through but also about those who abstain.

Consider a population of workers of either high or low ability in proportion $\alpha$ and $1 - \alpha$. Their respective productivities are $\theta_h$ and a lower $\theta_l$. If abilities were observable then competition between firms (zero profit condition) would drive differentiated wages to $w_h = \theta_h$ and $w_l = \theta_l$. More plausibly, ability is unobservable. Each worker can then go to university for a duration of $q$ months in order to signal itself to future employers. The disutility of schooling (sic) is $c(q, \theta) = \frac{q}{\theta}$ i.e., both cost and marginal cost are lower for the high ability worker. Notice that education does not increase productivity, it only serves a signal of ability. Outside opportunities are normalized to zero.

Firms compete for workers by offering a wage scheme $w(q)$. In the absence of the educational system the market wage is equal to the average productivity $E[\theta] = \alpha \theta_h + (1 - \alpha) \theta_l$. When a worker shows up with a level of education $q$, a firm’s belief that he is of high ability is $\alpha q$. Its rational wage offer is thus $\hat{w}(q) = \alpha q \theta_h + (1 - \alpha q) \theta_l$.

In a pooling equilibrium all workers choose the same level of education $\tilde{q}$ thus the belief after seeing $\tilde{q}$ remains $\alpha$ so that the equilibrium wage is $E[\theta]$. The belief after seeing $q \neq \tilde{q}$ can be $\alpha_q = 0$ (if something that should not appear still appears then employers being prudent think that the worker is of the worst type) so that the optimal wage offer is $\theta_l$. Given this behavior, the best alternative to $\tilde{q}$ for workers is to maximize $\theta_l - \frac{q}{\theta}$ i.e., choose no education whatever the innate ability. The proposed equilibrium choice $\tilde{q}$ is thus optimal if $E[\theta] - \frac{\tilde{q}}{\theta} \geq \theta_l \Leftrightarrow \tilde{q} \leq \alpha \theta_h (\theta_h - \theta_l)$ for both $\theta_h$ and $\theta_l$. The condition is thus $\tilde{q} \leq \alpha \theta_h (\theta_h - \theta_l)$ in order that the least able workers accept to go to university. Obviously the efficient
equilibrium is \( \tilde{q} = 0 \) as it saves workers the cost of going to university.

In the separating equilibrium different types of workers chooses different level of education \( q_h \) and \( q_l \), thus they are identified and paid at their productivity i.e., \( \hat{w}(q_h) = \theta_h \) and \( \hat{w}(q_l) = \theta_l \). Beliefs following \( q_h \) and \( q_l \) are degenerate and for a different \( q \) we may set \( \alpha_q = 0 \). As for optimal schooling it is clear that whatever belief held by employers a low ability type cannot get a wage lesser than \( \theta_l \) thus he shall either pretend to be a high type by choosing \( q_h \) to get \( w = \theta_h \) or never go to school to avoid the disutility. Hence \( q_l = 0 \) and he will not masquerade if \( \theta_h - \frac{q_h}{\theta_l} \leq \theta_l \Leftrightarrow q_h \geq q_0 \equiv (\theta_h - \theta_l)\theta_l \). Clearly a high ability type that spends more than \( q_0 \) in education cannot be mistaken for a low type. Hence we get a class of equilibria where \( q_h \geq q_0 \) and \( q_l = 0 \). It is now clear that other beliefs agree with this outcome; for instance \( \alpha_q = 0 \) if \( q < q_h \) and \( \alpha_q = 1 \) if \( q \geq q_h \). Agents utilities in the efficient equilibrium where \( q_h = q_0 \) are \( u_l = \theta_l < u_h = \theta_h - \frac{(\theta_h - \theta_l)\theta_l}{\theta_h} \).

There exists equilibria where one type chooses one action while the other randomize between imitation and separation. For instance the high type chooses \( q_h \) while the low type imitates with probability \( \lambda \) and separates with \( q = 0 \) otherwise. The equilibrium belief is thus \( \alpha_\lambda = \frac{\alpha}{\alpha + (1-\alpha)\lambda} \) and the associated wage \( \hat{w}(q_h) = \alpha_\lambda \theta_h + (1 - \alpha_\lambda)\theta_l \). The low type plays a mixed strategy only if he is indifferent between \( q_h \) and 0 i.e., \( \hat{w}(q_h) - \frac{q_h}{\theta_l} = \theta_l \Leftrightarrow q_h = \tilde{q}_\lambda \equiv \frac{\alpha(\theta_h - \theta_l)\theta_l}{\alpha + (1-\alpha)\lambda} \leq q_0 \) showing that the efficient separating equilibrium is the limit of the class of mixed equilibrium as \( \lambda \) approaches zero. The off-equilibrium beliefs can be \( \alpha_q = 0 \) if \( q < \tilde{q}_\lambda \) and \( \alpha_q = \frac{\alpha}{\alpha + (1-\alpha)\lambda} \) if \( q > \tilde{q}_\lambda \).

Advertising

Kihlstrom and Riordan (1984) show that advertising can be a signal of quality in an oligopoly setting if a high quality producer is patient enough.

The problem faced by a high quality producer is that in the absence of a cheap and reliable certification agency, it is not able to inform correctly potential customers about the quality of its products. For such experience goods, the opinion of consumers is either confirmed or revised upward or downward according to what they bought and what they anticipated. The following “hit-and-run” strategy immediately comes to mind to take advantage of this delay in the revelation of true quality: package a cheap and low quality product as if it was
top-notch, sell it at a high price for one period and then lower the price since nobody will get fooled now that the word has spread regarding what was the real quality inside the gleaming package.

Let $\pi_{ij}$ denote the profit during one period made by selling quality $i$ when consumer think it is quality $j$. We obviously have the ranking $\pi_{lh} > \pi_{hh}$ since cost are lower for a lower true quality but also $\pi_{lh} > \pi_{ll}$ since consumers are ready to pay more for (what they think is) a higher quality. The present value of aggregated profits over many periods for the “hit-and-run” strategy is $\pi_{lh} + \frac{1}{r}\pi_{ll}$ where $r$ is the interest rate\(^5\) and is obviously greater than the profit $\pi_{ll} + \frac{1}{r}\pi_{ll}$ gained by a truthful maker of a low quality product.

To force out the deceptive strategy of “hit-and-run”, a high quality producer can simply spend an amount of advertising $A$ larger than the difference $\pi_{lh} - \pi_{ll}$ because it will nullify the benefit of mimicking by a low quality producer. Yet the high quality producer should take care of not spending too much in advertising because he can still change side and switch to the low quality product.\(^6\) The latter condition reads $\pi_{hh} - A + \frac{1}{r}\pi_{hh} \geq \pi_{ll} + \frac{1}{r}\pi_{ll} \Leftrightarrow A < (\pi_{hh} - \pi_{ll}) \frac{1}{1 + r}$. A first necessary condition for advertising to be a signal of quality is $\pi_{hh} > \pi_{ll}$ i.e., high quality products must generate more profits than low quality ones in a perfectly informed market. As shown in §??, the condition is likely to be satisfied.

Comparing the lower and upper-bound for $A$ we derive a second necessary condition for advertising to be a signal of quality: $r < \frac{\pi_{hh} - \pi_{ll}}{\pi_{lh} - \pi_{hh}}$ i.e., the producer is patient and cares mostly for future profits. The numerator is the truthful profit difference between the qualities while the denominator is the cost difference between the qualities.

### 3.2 Screening & Self-Selection

The problematic of information acquisition is referred to as screening. For instance, §?? shows a monopoly (or a firm with market power) trying to extract consumer surplus from its clients (agents). The valuable private information of agents is their willingness to pay for the good sold by the principal. When direct discrimination is unavailable, the firm resorts to indirect discrimination and de-

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\(^5\)The net present value of future payments of $\text{1}\,\text{€}$ per period starting tomorrow is $1/r$.

\(^6\)This is obviously an extreme assumption only used for its simplicity.
signs a menu of contracts that is offered to all customers and out of which each type of client picks his preferred option, a procedure called “Self-Selection”. This way, the monopolist succeeds to obtain the private information although not completely since more interesting types (high WTP) ends up with an information rent.

This section presents cases of procurement, monopolistic and regulatory screening. Models are presented in order of difficulty so that each builds heavily on the previous one. In the first case, a principal wants to procure a service from an agent at the lowest possible cost. In the second situation, known as non linear pricing, a firm holding market power discriminate among her clients with a variety of contracts to maximizes profit. The third model studies a regulator trying to elicit cost information from the regulated firm in order to apply an efficient pricing scheme and maximize welfare. Lastly, we present the original insurance problem of “adverse selection”. When insurers compete perfectly, it is possible to pick up the best clients (aka cream skimming) with a well designed proposal. This may force firms left with risky clients to go out of business so that a market failure occurs.

3.2.1 Procurement

Consider a firm procuring refuse collection to a city or a specialized firm procuring IT services to a manufacturer. According to whether the technology is cutting-edge or obsolete, a quite different outcome is called for. It would be useful for the principal to know the agent’s cost in order to be able to fine tune the volume and price of the service. Yet, this information is often private knowledge to the agent; the principal thus faces a problem of “information revelation” and must “screen” the agent to uncover it.

**Symmetric Information** The principal’s valuation for output \( q \) is \( V(q) \) while her WTP for an additional unit is \( P(q) = V_m(q) \). The agent’s marginal cost \( \theta \) is either low \( (\theta_l = c) \) or high \( (\theta_h = c + \delta) \). This parameter which synthesizes the private information problem here is referred to as the *type* of the agent; in our simple polar example we have the *good* and *bad* types or the *cutting-edge* and *obsolete* technologies.
The objectives are $\pi = V(q) - t$ for the principal and $u = t - \theta q$ for the agent; their sum is the welfare $W(\theta, q) = V(q) - \theta q$. Money is thus simply a means to transfer utility among parties. Ex-ante, before the agent’s type is determined, we deal with the expected welfare

$$E[W] = \alpha \left[ V(q_l) - c q_l \right] + (1 - \alpha) \left[ V(q_h) - (c + \delta) q_h \right]$$ (3.7)

Pareto optimality calls for its maximization and can be achieved type-by-type with output $q^*_\theta = P^{-1}(\theta)$, the solution of $V_m = \theta$, for $\theta = l, h$.

If the principal knows the agent’s type $\theta$, he can offer him a contract $(q, t)$ with $t \geq \theta q$ in order to meet his participation constraint $u \geq 0$. Since the principal dislikes payments, it is optimal for her to saturate the previous constraint with $t = \theta q$. Profit is now $\pi = W$ and output remains the sole element to be chosen; its optimal level is the efficient one $q^*_\theta$. This so-called “first-best” or ideal scheme is discriminatory since different contracts are proposed to different types of agents. Note the similarity with the basic moral hazard case of §2.1.

**Asymmetric Information** When the principal ignores the firm’s type, she can offer an “attractive” contract guaranteeing full participation i.e., satisfying the participation constraint of any type. Because types are ordered, the condition $\{\forall \theta, t \geq \theta q\}$ boils down to $t \geq \theta h q$ i.e., the problem is bring the bad type on board. As before, it is optimal to saturate the latter so that the principal’s objective becomes $\pi = V(q) - \theta h q$. This is as if the agent was, for sure, of the worst type, in which case the optimal output is $q^*_h$. This crude solution completely forsakes the fact that, on average, the agent is much more efficient and that welfare (or principal’s profit) could be vastly improved.\(^7\)

The adequate way to manage the diversity of possible technologies (types) is to offer a menu of contracts and let each type of agent picks his most preferred one. For instance, the principal could offer simultaneously the ideal contracts $\gamma^*_l = (q^*_l, t^*_l)$ and $\gamma^*_h = (q^*_h, t^*_h)$, hoping that each type of agent will pick the one intended for him. The bad type will stick to $\gamma^*_h$ (over $\gamma^*_l$) since by construction we have $0 = t^*_h - \theta h q^*_h > t^*_l - \theta h q^*_l = -q^*_l(\theta h - \theta l)$; this is because $\gamma^*_l$ requires a heavy work load paid at less than the actual cost. Sadly, the good type will also prefer

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\(^7\)As in §?? for price discrimination, excluding inefficient agents in order to improve the performance of efficient ones is possible but if $V$ is large this is never optimal.
\( \gamma^*_h \) as \( 0 = t^*_i - \theta_l q^*_l < t^*_h - \theta_l q^*_h = q^*_h (\theta_h - \theta_l) \); this time, the workload is light but paid handsomely because from the point of view of the good type, he stands to make a profit equal to the marginal cost difference on every unit of output.

The question then is whether we can design a complex screening scheme that would lead to discriminating behavior so as to uncover the agent’s type and have him perform in a relatively efficient manner? The answer is twofold. Yes, we can discriminate (or screen) using only as many contracts as there are types provided they satisfy some incentive constraints. Yet, in the process, the principal has to give a rent to the good type and accept that the bad type under performs wrt. the ideal perfect information situation.

**Revelation Principle**

We tackle here the issue of complexity with the revelation principle. In order to motivate the agent to reveal his type (private information), the principal can offer a menu of contracts; for instance a red, a green, a yellow and a blue contracts. If type \( l \) picks the green one while type \( h \) picks the red one then it must be the case that type \( l \) prefers the green over the red while type \( h \) ranks them inversely. It is then obvious that the blue and yellow contracts were unnecessary in the first place. It is even useless to name contracts with colors; we can directly name them “low cost” and “high cost” (or use labels \( l \) and \( h \)). As shown by Myerson (1979), this intuition carries on to much more mathematically advanced models of asymmetric information.

In our simple two types settings, the revelation principle allows us to restrict attention to pairs of contracts that are *self-selecting* in the sense that each type of agent prefers the contract designated for him over any other. The conditions that can be deduced from these observations are called the *incentive compatibility* constraints (IC). Lastly, we must not forget the participation constraint i.e., the contract designed for a specific type of agent must leave him with a non negative profit for otherwise he would decline it. This yield a set of *individual rationality* constraints (IR).
Second Best Solution

Given a pair of contracts \((q_h, t_h)\) and \((q_l, t_l)\), the principal’s objective is to maximize expected profit

\[
\mathbb{E}[\Pi] = \alpha \left[ V(q_l) - t_l \right] + (1 - \alpha) \left[ V(q_h) - t_h \right]
\]  

under constraints

\[
\begin{aligned}
& (IC) \left\{ 
\begin{aligned}
& t_l - \theta_l q_l \geq t_h - \theta_l q_h & \quad \text{(1)} \\
& t_h - \theta_h q_h \geq t_l - \theta_h q_l & \quad \text{(2)}
\end{aligned}
\right. \\
& \text{and} \quad (IR) \left\{ 
\begin{aligned}
& t_l \geq \theta_l q_l & \quad \text{(3)} \\
& t_h \geq \theta_h q_h & \quad \text{(4)}
\end{aligned}
\right.
\]

The (IC) constraints simplify into \(\theta_l(q_l - q_h) \leq t_l - t_h \leq \theta_h(q_l - q_h) \Rightarrow 0 \leq (\theta_h - \theta_l)(q_l - q_h) \Rightarrow q_l \geq q_h\) since \(\theta_h > \theta_l\). We also derive \(t_l \geq t_h\). Observe now that (4) and (1) imply (3) since \(t_l - \theta_l q_l \geq t_h - \theta_l q_h > t_h - \theta_h q_h \geq 0\). Neglecting (2) for the moment,\(^8\) the optimum satisfies \(t_h = \theta_h q_h = (c + \delta) q_h\) (no rent for the high cost agent) and \(t_l = t_h + \theta_l(q_l - q_h) = c q_l + \delta q_h\) (minimum rent for the low cost agent). We may then rewrite the expected profit as

\[
\mathbb{E}[\Pi] = \alpha \left[ V(q_l) - c q_l \right] + (1 - \alpha) \left[ V(q_h) - (c + \delta) q_h \right] - \alpha \delta q_h
\]

We see that wrt. the welfare objective of (3.7), an added cost appears; it is the information rent left to the efficient type to motivate him to reveal his private information (here, his ability to perform at low cost). The optimum in the incomplete information setting is called “second best” because the informational rent generates an allocative distortion as we now demonstrate.

The FOCs for maximizing (3.10) are \(P_l(q_l) = c\) and \(P_h = c + \frac{\delta}{1 - \alpha} > c + \delta\).

The first result is called “no distortion at the top” because the low cost firm (good type) is ordered to produce the efficient quantity. The high cost firm (bad type), on the other hand, is instructed to reduce output (and perform inefficiently) in order to lower the rent left to the good type. This distortion is meant to hurt a good type pretending to be a bad type, while not hurting the bad type as much. Thus, the principal is trading-off the efficient behavior of the low type

\(^8\)This technique can be illustrated as follows: who is the tallest Dutch? If we lack a Dutch database we can still search a European database and use our intuition that Dutchs are the tallest people of Europe. Once we encounter the tallest European, we only need to check that he/she is a Dutch citizen.
with the information rent of the high type. Lastly, we check that \( t_l - \theta_h q_l \leq q_l \geq q_h \).

### 3.2.2 Non Linear Pricing

Non linear pricing by a monopolists involves selling to consumers of unknown type while procurement involves purchasing goods or services from providers of unknown type. The optimization procedures are thus dual one of another.

#### Indirect Price Discrimination

A firm with marginal cost \( c \) sells a good or service under a package \((q, t)\) where \( q \) is consumption and \( t \) total price (fee). If potential clients form an homogeneous population sharing WTP \( P(.) \), the optimal discrimination scheme is to offer the efficient quantity \( q^* \) solving \( P(q) = c \) for total fee \( t^* = V(q^*) \) where \( V(q) \equiv \int_0^q P(x) \, dx \) is the gross utility enjoyed from consuming \( q \) units (cf. §??).

Consider now the differentiated “home” and “pro” market segments whereby the latter agree to pay premium \( \delta \) over the former i.e., we have WTPs \( P_l(.) = P(.) \) and \( P_h(.) = P(.) + \delta \) (and gross utility functions \( V_l \) and \( V_h \)).  

9 Notice that contrary to the previous section, the “good” type is \( h \). We may interpret the premium \( \delta > 0 \) as a measure of the information asymmetry affecting the monopolist who cannot distinguish among the two types.

By the revelation principle, the monopoly need only design two contracts \((q_h, t_h)\) and \((q_l, t_l)\) satisfying self-selection. Letting \( u_i = V_i(q_i) - t_i \) for \( i = l, h \), the per-type profit is \( \pi_i = t_i - cq_i = V_i(q_i) - cq_i - u_i \) (note that \( \pi_i + u_i \) is welfare). The individual rationality constraints (IR) are \( u_i \geq 0 \) while the incentive compatibility constraints (IC) are \( u_l \geq V_l(q_h) - t_h = u_h + \delta q_l \) and \( u_h \geq V_h(q_l) - t_l = u_l - \delta q_h \) which simplify into

\[
\delta q_l \leq u_h - u_l \leq \delta q_h \tag{3.11}
\]

from which we deduce \( q_h \geq q_l \) (in a manner quite similar to (3.9)).  

10 The information rent of the “pro” is again the lower bound \( \delta q_l \) in (3.11); it is the net

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9 The general model requires only that \( P_h(.) - P_l(.) > 0 \) is valid for the entire domain.

10 In the general case, the IC constraints are \( \int_0^{q_h} (P_h - P_l) \geq u_h - u_l \geq \int_0^{q_l} (P_h - P_l) \).
surplus he can secure by grabbing the proposal designed for a “homer” i.e., by pretending to be of a different type.

As in the procurement case, we come to the conclusion that the firm leaves no rent to a “homer” (i.e., sets \( u_l = 0 \)) and leaves the minimum information rent to a “pro” (i.e., sets \( u_h = \delta q_l \)). The expected profit is thus very much like (3.10) with

\[
E[\Pi] = \alpha \pi_h + (1 - \alpha) \pi_l = \alpha \left[ V_h(q_h) - c q_h \right] + (1 - \alpha) \left[ V_l(q_l) - c q_l \right] - \alpha \delta q_l \quad (3.12)
\]

The FOCs are \( c = P_h = P(.) + \delta \), leading to choose the efficient quantity \( \hat{q}_h = q_h^* \), and \( P_l = P(.) = c + \frac{\alpha \delta}{1 - \alpha} \) leading to choose a reduced quantity \( \hat{q}_l < q_l^* \).\(^{11}\) The ratio \( \frac{\alpha \delta}{1 - \alpha} \) may be interpreted as the marginal cost of information acquisition.

Observe that if \( \alpha \) and \( \delta \) are large then \( P_l(0) < \frac{\alpha \delta}{1 - \alpha} \) might become true. In that case, it is better to exclude “homers” by setting \( q_l = 0 \). The monopolist does this to avoid paying the information rent to “pros”; recall indeed that if the proposal is unappealing to “homers”, the firm can extract all the consumer surplus from “pros” because she faces a unique segment of homogeneous people.

### 3.2.3 Public Firm Regulation

As explained in §??, the ideal (second best) regulation for a public service is a per unit price close to marginal cost together with a subscription large enough to cover fixed cost. Choosing these two parameters is not an easy task for the regulator because she can only use the cost report handed over by the firm who might therefore have an incentive to overstate true cost. This possibility is a real issue given the better knowledge of the production technology. Combined with the fact that public funds involve a loss of efficiency (cf. §??), the optimal regulatory policy will be distorted as if there was an additional marginal cost of eliciting the private information of the firm.

Another issue affecting regulation is the cyclicity of demand of many public services. The existence of peaks and off-peak periods warrants a flexible price policy from an efficiency point of view. For reasons better explained in §??, inter-temporal discrimination is often forbidden and leads to excess capacity

\(^{11}\)In the general case, the equation is \((1 - \alpha)(P_l - c) = \alpha(P_h - P_l) \Leftrightarrow P_l = \hat{c} \equiv (1 - \alpha)c + \alpha P_h > c.\)
and large deadweight losses. In a few cases such as the pricing of highways operated under franchise, regulators are more and more willing to adapt their regulatory framework to account for this reality. Yet they face the problem of eliciting the demand from the operator in order to set the price at the efficient level. This is what we explore in the first part before delving into the more involved case of unknown cost.

Private Information on Demand

To assess the variability of demand, we simply assume that the market size parameter $a$ in the usual demand formulation $D(p) = a - bp$ is a random variable observed only by the regulated firm, thus unknown to the regulator. The firm has to build a capacity of service $k$ (at marginal cost $\delta$) and can produce up to $k$ units at marginal cost $c$.

Efficient Pricing The efficient pricing rule is found by following a simple procedure. Try first to price at marginal cost; if the resulting demand can be met ($D(c) \leq k$) this is it; if on the contrary there is potential congestion ($D(c) > k$), then raise the price until excess demand vanishes ($D(p_k) = k$) i.e., in our example, set $p_k = \frac{a-k}{b}$. This is the most efficient manner to ration consumers i.e., that which minimizes the welfare loss generated by the limited service capacity (cf. §?? on congestion). The threshold market size such that marginal cost pricing does not generate congestion is $a_k \equiv k + bc$. We thus obtain $p_k(a) \equiv \min\{c, \frac{a-k}{b}\} = \begin{cases} c & \text{if } a < a_k \\ \frac{a-k}{b} & \text{if } a \geq a_k \end{cases}$.

The fact that the efficient price is a rule depending on some piece of information unknown to the regulator poses a problem for he has to trust the firm to reveal it correctly. However this is not going to happen because the firm has an incentive to overstate the true demand. Indeed, if the firm claims that the demand parameter is some $\hat{a}$ greater than the true level $a$, then the regulator will allow the price $p_k(\hat{a})$; it then remains to adjust $\hat{a}$ so as to equate $p_k(\hat{a})$ with the monopoly price.\(^{12}\)

\(^{12}\)If $k \geq \frac{a-bc}{2}$, the capacity is large enough to meet demand at the monopoly price $p^M(a) = \frac{a+bc}{2b}$; the announcement $\hat{a} = k + \frac{a+bc}{2} > a$ does the trick. If demand is so large that there is congestion at the monopoly price, the firm simply tells the truth and sells all her capacity at the
Fortunately, Riordan (1984) displays a simple contract restoring the incentives of the firm toward the common good: the firm is allowed to set the price $p$, has to meet demand up to capacity and receives subsidy $s_k(p) \equiv \delta k - (p - c)k$, contingent on her announced price.\(^{13}\) The scheme involves a fixed subsidy covering the capacity cost and a variable tax component to extract any margin made over sales. The firm’s profit is now $\pi(p) = (p - c) \min \{D(p), k\} - \delta k + s_k(p) \leq 0$ by the very definition of $s_k(.)$. Hence, the most that she can expect is to earn zero (extraordinary) profit. We now show that $p_k(.)$ is a rule that precisely enables to achieve this maximum. Indeed, the firm earns:

$$\pi(p_k) = \begin{cases} (c - c)D - \delta k + (c + \delta - c)k &= 0 \quad \text{if } a < a_k \\ (p_k - c)k - \delta k + (c + \delta - p_k)k &= 0 \quad \text{if } a \geq a_k \end{cases}$$

We conclude that the optimal regulatory mechanism is akin to a two-part tariff whereby both the subsidy and the unit price respond to shifting demand conditions.

Efficient Capacity The efficient capacity is found by equating the marginal cost $\delta$ of capacity expansion to its marginal value which is the wedge between the WTP of the first rationed consumer (i.e., the price) and the marginal cost of service $c$. Since the price varies with the intensity of demand, the efficient capacity $k^*$ solves $E[p_k(.)] = c + \delta$.

When the regulator offers the price contingent subsidy $s_k(.)$, the firm prices efficiently and thus reveals the state of demand. The regulator is thus able to compute the expectation $E[p_k(.)]$ to ascertain whether the actual capacity is too small or too large.

Once the firm has been instructed to invest efficiently at the level $k^*$, the subsidy mechanism becomes budget balanced; indeed, the expected subsidy is $E[s_k(.)] = k(c + \delta - E[p_k(.)])$ since the firm is motivated to price efficiently. This expression is zero precisely when the efficient capacity $k^*$ is build. This is a very convenient property since deficit or surplus are always source of costly haggling.

\(^{13}\)Formally we also have to specify a zero subsidy if the price is set below marginal cost in order to force loss should the firm ever choose to price that way.
among stake-holders in the regulation process.

**Private Information on Costs**

When the regulated firm knows better than the regulator its ability to perform the job, we have a situation similar to procurement. The only difference is the principal’s objective: a regulator cares for both consumer surplus and firm profit but is reluctant to tax the rest of the economy to finance the firm. The marginal cost of public $\lambda$ funds alluded to in §?? will introduce a slight modification of the optimal second-best regulatory policy.\textsuperscript{14} We follow Laffont and Tirole (1986) and Baron and Myerson (1982).

The procurement model for variable output seen in §3.2.1 is easily adapted. The regulated firm’s profit is $\pi = t - \theta q$ where the marginal cost is $\theta_l = c$ with probability $\alpha$ or $\theta_h = c + \delta$ with complementary probability. Raw consumer surplus is $U(q)$ (with WTP $P(.)$) while a contract is an output-transfer pair $(q, t)$. The regulator’s objective is welfare net of the cost of public funds i.e.,

$$W = U(q) - \theta q - \lambda t = U(q) - (1 + \lambda)\theta q - \lambda\pi \propto V(q) - \theta q - \tau\pi \quad (3.13)$$

with $V(q) \equiv \frac{U(q)}{1 + \lambda}$ and $\tau \equiv \frac{\lambda}{1 + \lambda}$.

The (IC) and (IR) constraints are identical to (3.9) leading to an information rent $\tau\delta q_h$ once transfers are minimized. Expected welfare is thus

$$\mathbb{E}[W] = \alpha \left[ V(q_l) - cq_l \right] + (1 - \alpha) \left[ V(q_h) - (c + \delta)q_h \right] - \alpha\tau\delta q_h \quad (3.14)$$

which is identical to (3.10) except for the multiplier $\tau$ in the last term.

The FOC for the low cost firm is $V_m = c \Leftrightarrow P(.) = (1 + \lambda)c$ i.e., she produces the efficient quantity (*no distortion at the top*). The FOC for the high cost firm is $P(.) = (1 + \lambda)(c + \delta) + \frac{\alpha\lambda\delta}{1 - \alpha}$ i.e., equalizes marginal benefit with a *virtual* marginal cost summing the true marginal cost of production and the cost of eliciting the firm’s private information.\textsuperscript{15} Although the regulator does not care per-se for wealth distribution, he tries to avoid distortionary tax collection and is thus ea-

\textsuperscript{14}In the present context, $\lambda$ is an ad-hoc parameter while in the general theory, its value is obtained (computed) at the equilibrium. Alternatively, one can assume that the regulator has a strict preference for consumer surplus over firm rent.

\textsuperscript{15}Note that $(IC_h)$ is satisfied since it reads $0 \geq \delta(\hat{q}_h - \hat{q}_l)$ and we saw that $\hat{q}_h < q^*_h < q^*_l = \hat{q}_l$. 

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ger to minimize the information rent left to the low cost firm. Consequently, the
dearer firm is instructed to produce less than what would be efficient to avoid
imitation by a cheap firm \((\hat{q}_h < q^*_h)\).

### 3.2.4 Procurement and Moral Hazard

In many instances of procurement, the principal (e.g., government, firm) con-
tracts the agent (e.g., builder, maker, consultant) to produce a single and id-
iosyncratic item such as a bridge, an industrial design or a firm re-organization.
In that case, the principal would like the agent to specialize over that task and
invest to develop specific skills that would reduce the overall cost the project.

As before, the agent’s basic technology can be cutting-edge or obsolete, an
information known only to him. The novelty here is the possibility to meliorate
the technology (reduce cost) by investing into assets or capital specific to the
delegated task (cf. §2.2).

Denote \(\beta\) the cost saving undertaken be the agent under a regime of resid-
ual claimancy (cf. §2.1).\(^{16}\) This investment, however, is not contractible as it
involves personal and unobservable skills and actions. This element of moral
hazard (cf. §2) will generate an adverse selection problem.

**Worst Case: C+ and FP**

Under a Cost-Plus Contract (C+), the principal repays the agent all his expenses
(related to the task). The latter seeing that any improvement effort falls on his
shoulders will undertake none (cf. §2.1.2). The principal thus reimburses the
full cost \(\theta\) and on expectation pays \(\hat{\theta} \equiv \mathbb{E}[\theta]\).

Under a fixed price (FP) contract, the principal offers the fixed price \(t\) for
completion of the item. The agent, whatever his type, then becomes residual
claimant of any technology melioration thus invest optimally and saves \(\beta\). His
payoff is then \(t - \theta + \beta\). He will accept the initial offer only if \(t \geq \theta - \beta\). If the
principal insists on getting the item no-matter-what, she has to set a high price

\(^{16}\) In a continuous setting, the opportunity value to the agent of reducing the item cost by \(e\) is
\(d(e)\) where \(d\) is increasing convex. Let \(e^*\) be the efficient effort solving \(1 = \dot{d}\) and \(\beta \equiv e^* - d(e^*)\)
be the maximal technology saving.
to attract the high cost firm (bad type) i.e., \( t \geq \theta_h - \beta \) (with equality at the optimum). In that case, the low cost firm (good type) enjoys a large rent.

This “no risk FP” contract is better than C+ if \( \theta_h - \beta \leq \bar{\theta} \Leftrightarrow \beta \geq \alpha \delta \) i.e., when the gain from optimized technology is large, so that providing good incentives is important. Conversely, if the information asymmetry is large (high \( \delta \)) then C+ is better to avoid leaving such a large rent to the low cost agent.

**Middle Case: a simple option**

A frequently observed contract is a C+ with an option to switch to FP. Obviously, if the fixed price \( t \) is low, no one ever picks the option so we are back to the pure C+ case. Likewise, if \( t \) is large, everyone picks the FP option and we are back to the “no risk FP” case. For intermediate \( t \), only the low cost agent picks the FP option and implements the maximal cost saving \( \beta \). Among these, \( t = \theta_l - \beta \) is optimal as it minimizes the payment. The expected cost for the principal is thus \( \alpha(\theta_l - \beta) + (1-\alpha)\theta_h = \bar{\theta} - \alpha \beta \) which clearly dominates the C+ contract. The practical value of this simple option contract is that it can be computed without any knowledge of the agent’s ability to improve his technology i.e., without resorting to a screening mechanism.

**Best Case: no information asymmetry**

To develop fully the model we follow footnote 16 and denote \( e \) a marginal cost reduction and \( d(e) \) the financial cost of achieving it. The principal’s objective is to minimize the expected cost

\[
E[C] = \alpha [\theta_l - e_l + d(e_l)] + (1-\alpha) [\theta_h - e_h + d(e_h)]
\]

(3.15)

where \( e_i \) is the level of cost savings implemented by the agent (which depends on the contract he agreed to).

If the principal could observe the agent’s type, he could taylor a FP contract for each type \( i = l, h \) with \( t^*_i = \theta_i - \beta \). This offer would be accepted and followed by implementation of the efficient cost savings so that realized cost would be \( \theta_i - \beta \). The principal would thus get the item done for an expected cost of \( E[C] = \bar{\theta} - \beta \) i.e., the maximal improvement over the C+ base case. Equivalently, the
principal offers the agent to bring cost down to $\theta_i - \beta$ and pays him exactly that amount.

In the realistic case where the principal does not observe the agent’s type, she has to devise a revelation mechanism. We already know from the previous models that naively offering the efficient FP contracts $t^*_h$ and $t^*_l$ fails because both types prefer the greater payment $t^*_h$ i.e., the outcome is the same as with the “no risk FP” contract.

Second Best: complex option

Our basic instrument is the target contract $(c, t)$ whereby the principal agrees to pay the agent $t + c$ if the cost $\theta - e$ is brought down to $c$. When type $\theta$ accepts such an offer, he must implement a minimum effort saving $e = \theta - c$ to get the payment.\(^{17}\) His profit (rent) is then $\pi(\theta, c, t) = t - d(\theta - c)$ since procurement expenses are paid by the principal.

The principal offers contracts $(c_l, t_l)$ and $(c_h, t_h)$. We denote $e_i = \theta_i - c_i$ for $i = l, h$ and $\pi_i = \pi(\theta_i, c_i, t_i)$. The (IC) and (IR) constraints are then $\pi_i \geq \pi(\theta_i, c_j, t_j)$ and $\pi_i \geq 0$ for $i = l, h$ and $j \neq i$. With target contracts, if the low cost agent picks contract $(c_h, t_h)$, he will have to exert effort $\theta_l - c_h = e_h - \delta < e_h$ because he is intrinsically more efficient than the high cost agent. On the contrary, if the high cost agent picks $(c_l, t_l)$, he will be forced to exert effort $\theta_h - c_l = e_l + \delta > e_l$ since it is more difficult for him to pretend to be a low cost agent.

The principal’s objective is to minimize the expected cost of producing the item

$$E[C] = E[c_i + t_i] = E[\theta_i - e_i + d(e_i) + \pi_i]$$

under the (IC) and (IR) constraints

\[
\begin{align*}
&\left\{ t_l - d(e_l) \geq t_h - d(e_h - \delta) \right\} (IC_l) \\
&\left\{ t_h - d(e_h) \geq t_l - d(e_l + \delta) \right\} (IC_h)
\end{align*}
\]

and

\[
\begin{align*}
&\left\{ t_l \geq d(e_l) \right\} (IR_l) \\
&\left\{ t_l \geq d(e_l) \right\} (IR_h)
\end{align*}
\]

As before, the (IC) conditions simplify to $d(e_l + \delta) - d(e_h) \geq t_l - t_h \geq d(e_l) - d(e_h - \delta) \equiv \nu$ which is the information rent that the low cost agent receives to reveal his type. Minimizing rents by setting $\pi_h = 0$ and $\pi_l = \nu$, the objective

\[17\text{He won't however do more because he supports the financial cost } d(e).\]
E[C] = \alpha [\theta_l - e_l + d(e_l)] + (1 - \alpha) [\theta_h - e_h + d(e_h)] + \alpha [d(e_h) - d(e_h - \delta)] (3.18)

which is the complete information formula (3.15) plus the cost of information acquisition, the informational rent paid to the low cost firm (good type). The FOCs are

\[ d'(e_l) = 1 \quad \text{and} \quad d'(e_h) = 1 - \frac{\alpha}{1 - \alpha} \left( d'(e_h) - d'(e_h - \delta) \right) \quad (3.19) \]

leading to optimal choices \( e_l = e^* \) and \( e_h < e^* \) i.e., no distortion at the top and an optimal reduction of effort at the bottom to reduce the information rent. Lastly, we need to check that \((IC_h)\) is satisfied at the candidate optimum. The condition reads

\[ \pi_h = 0 \geq t_l - d(e_l + \delta) \Leftrightarrow d(e_h) - d(e_h - \delta) \leq d(e_l + \delta) - d(e_l) \]

which is true because we assumed \( d \) convex and we have \( e_h < e_l \) at the optimum.

This complex scheme improves over the simple C+&FP option but not so much as we now proceed to show.

**Extension to many types †**

To compare the second-best optimal self-selecting menu of contracts with the simple C+&FP option, Rogerson (2003) assumes that the agent’s type is uniformly distributed between \( \theta_l \) and \( \theta_h \). We also take \( d(e) = \frac{e^2}{4\beta} \) in order that \( e^* = 2\beta \) and that \( \beta \) is indeed the maximum of \( e - d(e) \).

A C+&FP option at \( t \) leads all types below \( \hat{\theta} = t + \beta \) to pick the FP option and all the types above to stick to C+. The contract is thus parametrized by \( t \) or \( \hat{\theta} \). The expected cost is then \( C(\hat{\theta}) = \int_{\theta_l}^{\hat{\theta}} (\hat{\theta} - \beta) dF(x) + \int_{\theta_l}^{\theta_h} x dF(x) \) and its derivative is proportional to \( f(\hat{\theta})(\hat{\theta} - \beta - \hat{\theta}) + F(\hat{\theta}) \). With the uniform distribution, the equation becomes \( \hat{\theta} - \theta_l = \frac{\beta}{\delta} \) and the optimal cutoff level is thus \( \hat{\theta} = \min\{\theta_h, \theta_l + \beta\} = \theta_l + \min\{\delta, \beta\} \). The optimal optional FP is then \( t^* = \theta_l + \min\{0, \delta - \beta\} \). The expected cost is \( \hat{C} = \hat{\theta} - \min\{\beta - \frac{\delta}{2}, \frac{\beta^2}{2\delta}\} \). The percentage improvement of the simple option over the basic C+ in terms of the maximum cost saving \( k \) is then \( \hat{\rho} \equiv \frac{\hat{\theta} - \hat{C}}{\beta} = \min\{1 - \frac{\delta}{2\beta}, \frac{\beta}{2\delta}\} \).

Under the optimal second-best contract, the principal offers a menu \((c(\theta), t(\theta))\) for \( \theta \in [\theta_l, \theta_h] \). We let \( e(\theta) = \theta - c(\theta) \) and \( \pi(\theta) = t(\theta) - d(e(\theta)) \). The IC condition
is \( \dot{\pi}(\theta) = -\dot{d}(e(\theta)) \) while the expected cost is \( \mathbb{E}[\theta - e(\theta) + d(e(\theta)) + \pi(\theta)] \). The FOC for \( \theta \) is then \( \ddot{d}(\theta) = 1 - \frac{F(\theta)}{f(\theta)} \dot{d}(\theta) \). In our uniform distribution example, it reads \( \frac{e}{2\beta} = 1 - \frac{\theta - \theta_l}{2\beta} \) leading to \( e(\theta) = \min\{0, 2\beta + \theta_l - \theta\} \). Finally Rogerson (2003) shows that \( \rho^* = \min\left\{1 - \frac{\delta}{2\beta} + \frac{\delta^2}{12\beta^2}, \frac{2\beta}{3\delta}\right\} \).

The ratio of \( \hat{\rho} \) to \( \rho^* \) is always greater than \( \frac{3}{4} \) and equal to this value for \( \delta \geq 2\beta \). This means that a simple C+ contract with a fixed price option computed without knowledge of the agent’s preferences is within 75% of the second-best efficiency frontier.

**Regulation †**

Laffont and Tirole (1986) studies regulation of a firm whose underlying technology is private information and which is furthermore able to implement cost savings i.e., a situation where the adverse selection problem (private information) mingles with moral hazard (cf. also Laffont and Tirole (1993)).

The regulator is able to uncover marginal cost using accounting statements and thus can instruct the firm to produce efficiently conditional on the realized cost. However, the latter may be high when in fact the firm would have been able to meliorate its technology. The regulator’s problem is that she is unable to disentangle the role of effort \( e \) from the underlying marginal cost \( \theta \) in the final unit cost \( c = \theta - e \). A low observed cost occurs only when luck and hard work are combined together while a high cost is a sure indication of low effort (and back luck). The identification problem lies with an intermediate observation that can be due to luck if \( \theta = \theta_l \) or hard work if \( \theta = \theta_h \).

The adverse selection formulation follows §3.2.3 and adds a moral hazard issue as in §3.2.4. Welfare is thus a combination of (3.13) and (3.16) with \( W = V(q) - (\theta - e)q - d(e) - \lambda t \). Since profit is \( \pi = t - (\theta - e)q - d(e) \), the expected welfare

\[
\mathbb{E}[W] = \alpha \left[ V(q_l) - (1 + \lambda) (\theta_l - e_l)q_l + d(e_l) - \lambda \pi_l \right] + (1 - \alpha) \left[ V(q_h) - (1 + \lambda) (\theta_h - e_h)q_h + d(e_h) - \lambda \pi_h \right]
\]

is to be maximized under the (IC) and (IR) constraints (3.17). As in the previous
model, the rents are minimized so that (3.20) becomes

$$\mathbb{E}[W] = \alpha \left[ V(q_l) - (1 + \lambda) \left( (\theta_l - e_l) q_l + d(e_l) \right) - \lambda (d(e_h) - d(e_h - \delta)) \right] + (1 - \alpha) \left[ V(q_h) - (1 + \lambda) \left( (\theta_h - e_h) q_h + d(e_h) \right) \right]$$

(3.21)

Observe, as claimed initially, that the FOC for quantity is $P = (1 + \lambda)(\theta - e)$ i.e., production is efficient conditional on the achieved level of marginal cost. As for moral hazard, the effort of the cheap firm is the efficient level $e^*_l$ since it solves $d'(e_l) = q_l$ but the dear firm’s FOC is $d'(e_h) = q_h - \frac{\lambda \alpha}{(1 + \lambda)(1 - \alpha)} (d'(e_h) - d'(e_h - \delta))$ so that $\hat{e}_h < e^*_h$. There is an indirect effect on production i.e., $\hat{q}_h < q^*_h$, since a lesser effort raises the marginal cost while for the low cost firm, we have $\hat{e}_l = e^*_l$ and thus $\hat{q}_l = q^*_l$.

### 3.2.5 Insurance Cream Skimming

We present here a simplified version of Rothschild and Stiglitz (1976). Consider insurance against economic losses $c$ due to an unexpected event like a fire that forces a firm to stop production or an accident stopping an individual from working. The underlying risk for each client that seeks insurance is different, thus the insurer will try to gather a maximum amount of information to assess each riskiness. Yet some differences among clients remain unobservable. There are *safe* customers whose accident probability is $\theta_l$ and *unsafe* ones whose accident probability is a greater $\theta_h$. A policy consists of a premium $p$ and a reimbursement $d$ in case of damages. The expected utility of a type $\theta$ customer is

$$U(q, c) = \theta u \left( d - c - p \right) + (1 - \theta) u(-p)$$

(3.22)

where $u$ is increasing concave since customers are all risk averse (otherwise they wouldn’t seek insurance). The per capita profit of a risk neutral insurer is $\pi = p - \theta d$.

Pareto efficiency commands to eliminate risk by setting $d = c$. This efficient risk sharing outcome is achieved by competitive insurance markets insofar as risk classes are correctly identified by insurers; indeed, free entry and perfect competition drive economic profit to zero i.e., to actuarially fair premiums $p = \theta c$. 

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As soon as the different types of agents cannot be distinguished, the insurance market breaks down; more precisely, there cannot exist a pooling equilibrium where the same contract \( \gamma = (p, d) \) is bought by all types of agents. The underlying reason is that the average insurance cost, \( \mathbb{E}[\theta] \), stands between the real values \( \theta_l \) and \( \theta_h \) meaning that safe customers generate profits that can be used to cover the losses generated by unsafe customers.

Cream skimming is the process whereby a competing insurer succeeds to attract profitable safe customers and woo away the unprofitable unsafe ones. The trick is to offer a deductible increase \( \Delta d < 0 \) i.e., force customers to assume more risk, together with a premium reduction \( \Delta p < 0 \) that compensate safe risk customers only. To prove this claim, we compute

\[
\Delta U(\theta) = \theta u'(d - c - p) \Delta d - \left[ \theta u'(d - c - p) + (1 - \theta) u'(-p) \right] \Delta p
\]

\[
\propto \Delta d - \Delta p - \frac{1 - \theta}{\theta} \frac{u'(-p)}{u'(d - c - p)} \Delta p
\]

\[
\Rightarrow \frac{\Delta U(\theta)}{\Delta d} \propto 1 - \frac{\Delta p}{\Delta d} \left[ 1 + \frac{1 - \theta}{\theta} \frac{u'(-p)}{u'(d - c - p)} \right]
\]

(3.23)

and observe that \( \Delta U(\theta) < 0 \) for a small ratio \( \frac{\Delta p}{\Delta d} \) while it is positive for a large ratio \( \frac{\Delta p}{\Delta d} \). Hence we may adjust \( \frac{\Delta p}{\Delta d} \) so that \( \Delta U(\theta_l) = 0 \) holds. Now, observe that the last term in the bracket decreases with the risk index \( \theta \), thus \( \Delta U(\theta_h) < 0 \) i.e., the conditions for cream skimming hold true. The change in the contract leaves a low risk indifferent while repelling a high risk.

For the original \( \gamma \) to be a candidate equilibrium, it must not generate losses, thus \( p - \mathbb{E}[\theta]d \geq 0 \) so that \( p - \theta_l d > 0 \). We can now choose \( \Delta d \) small enough to guarantee that the insurer offering the altered contract earns the positive payoff

\[
p + \Delta p - \theta_l (d + \Delta d) = p - \theta_l d + \left( \frac{\Delta p}{\Delta d} - \theta_l \right) \Delta d > 0
\]

(whatever the sign of \( \frac{\Delta p}{\Delta d} - \theta_l \) since \( \Delta d \) is small). We have thus shown that cream-skimming is at work with this alteration of the original pooling contract that simultaneously attract low risks, repel high risks and generate profits.
Separation of Risks

The equilibrium is thus separating i.e., insurers offers menus of contracts and different risks elicit different contracts that identifies them afterwards. Since in our simple setting there are only two classes of risk or types, insurers need only offer two contracts \( \gamma_l = (p_l, d_l) \) and \( \gamma_h = (p_h, d_h) \) that will be picked up by the safe and unsafe customers respectively (cf. revelation principle seen in §3.2.1). Technically those incentive compatibility (IC) conditions are \( U(\theta_l, \gamma_l) \geq U(\theta_l, \gamma_h) \) and \( U(\theta_h, \gamma_h) \geq U(\theta_h, \gamma_l) \). Furthermore competition among insurers guarantees zero per-capita profits i.e., \( p_l = \theta_l d_l \) and \( p_h = \theta_h d_h \).

The unsafe type IC condition \( U(\theta_h, \gamma_h) \geq U(\theta_h, \gamma_l) \) reads

\[
\begin{align*}
    f(d_h) &\equiv \theta_h u((1-\theta_h)d_h - c) + (1-\theta_h)u(-\theta_h d_h) \\
    &\geq g(d_l) \equiv \theta_h u((1-\theta_l)d_l - c) + (1-\theta_h)u(-\theta_l d_l)
\end{align*}
\]

As \( u \) is concave, \( f' \propto u'(-p_h - c + d_h) - u'(-p_h) > 0 \) thus (3.24) is more easily satisfied by increasing \( d_h \). This means that the Bertrand type competition among insurers\(^\text{18}\) will drive \( d_h \) to \( c \) i.e., unsafe types receive full insurance and obtain a utility \( f(c) = u(-\theta_h c) \). Likewise competition tends to increase \( d_l \) but (3.24) is violated at \( d_l = c \) since everybody prefers the low premium \( p_l \) when both contracts give full insurance. Therefore the safe risk will have to support some risk (\( d_l < c \)). Since \( \theta_l < \theta_h \Rightarrow \theta_h (1-\theta_l) > (1-\theta_h)\theta_l \), we have

\[
\begin{align*}
    g'(d_l) &\equiv \theta_h (1-\theta_l) u'((1-\theta_l)d_l - c) - (1-\theta_h)\theta_l u'(-\theta_l d_l) \\
    &> (1-\theta_h)\theta_l \left[ u'((1-\theta_l)d_l - \bar{d}) - u'(-\theta_l d_l) \right] > 0
\end{align*}
\]

Having proved that \( g \) is increasing, there exists \( d^* < c \) such that \( g(d^*) = u(-\theta_h c) \) so that (3.24) is satisfied for any reimbursement \( d_l \leq d^* \). The benchmark reimbursement \( d^* \) leaves the unsafe risk indifferent between revelation and imitation. Once again competition among insurers drives \( d_l \) to its upper limit \( d^* \), so that \( \gamma_l = (\theta_l d^*, d^*) \). Lastly, we must check the incentive condition for the safe

\(^{18}\)By giving more risk sharing to bad types an insurer improves efficiency thus he can improve its per-capita profit over bad types. Imitation by competing insurers yields the result.
type: \( U(\theta_l, \gamma_h) \leq U(\theta_l, \gamma_l) \iff \)

\[
\begin{align*}
  u(-\theta_h c) &= g(d^*) = \theta_h u((1-\theta_l)d^*-c) + (1-\theta_h)u(-\theta_l d^*) \\
  &\leq \theta_l u((1-\theta_l)d^*-c) + (1-\theta_l)u(-\theta_l d^*)
\end{align*}
\]

is true since the weight on the larger term \( u(-\theta_l d^*) \) is increased.

**Equilibrium**

What we have characterized is the optimal pair of separating contracts. An immediate observation is that a cream-skimming contract is a non-optimal separating contract thus it is not a candidate equilibrium. To know whether \( \gamma_l = (\theta_l d^*, d^*) \) and \( \gamma_h = (\theta_h c, c) \) form an equilibrium we must check that there does not exist a pooling contract \( \hat{\gamma} \) preferred by both types of risk.

Observe indeed that an insurer could offer \( \hat{\gamma} = (\hat{p}; c) \) where the premium is computed to make the safe risk indifferent i.e., \( U(\theta_l, \gamma_l) = u(-\hat{p}) \). Then the IC condition for the safe type tells us that unsafe risks would have a strict benefit in switching from \( \gamma_h \) to \( \hat{\gamma} \). The expected profit is then \( \hat{p} - (\lambda \theta_l + (1-\lambda)\theta_h) c \) and will be positive if the proportion \( \lambda \) of safe types in the population is larger than \( \frac{\theta_h - \hat{p}/c}{\theta_h - \theta_l} \). The reason behind the existence of this profitable deviation is that the proportion of unsafe risk being small, it is not worthwhile for insurance companies to seek separation of types because it forces the large majority of safe risk to support costly risk; hence any insurer can offer a Pareto improving trade (an insurance service) to all consumers even if this means losing money on the few unsafe risks that are around.

So, when \( \lambda \) is large, a pooling contract \( \hat{\gamma} \) can successfully attack the separating \((\gamma_l, \gamma_h)\) but it is itself attacked by a cream skimming contract which is not a candidate separating equilibrium. This circularity proves that there is no equilibrium. Still, the model of insurance competition should be completed because the reaction of an insurer whose customers are stolen by a competitor matters. When attacked by a cream skimming contract \( \tilde{\gamma} \), the “old” pooling contract \( \hat{\gamma} \) is withdrawn because the zero profit condition \( p - (a\theta_l + (1-a)\theta_h) d = 0 \) yield losses once the safe types are gone. But then the cream skimming contract \( \tilde{\gamma} \) has to serve all types and becomes itself vulnerable to a cream skimming attack. The same reasoning applies in case of a pooling attack over a separating contract.
To solve this inconsistency Wilson (1977) proposes to redesign the competition between insurers as a stage game where:

- Insurers simultaneously offer "old" contracts (pooling or separating).
- Insurers simultaneously offer "new" contracts.
- Insurers simultaneously withdraw "old" contracts if they wish to.
- Customers can sign any contract.

In this game, the equilibrium is the separating one characterized earlier for small $\lambda$; otherwise it is the pooling contract $\hat{\gamma}$ giving the highest level of utility to safe types. Indeed if $\lambda$ is large, a cream skimming attack against $\hat{\gamma}$ needs $\lambda = \theta_l < \frac{\Delta p}{\Delta d} < \beta = \theta_h$ hence generates a profit of $(\lambda \theta_l + (1 - \lambda)\theta_h) \Delta d - \Delta p < \lambda (\theta_l - \theta_h) \Delta d < 0$. 

Chapter 4

Auctions

Auctions are very competitive trading mechanisms, so much so that the textbook examples of “perfectly competitive” markets are often the colourful markets for fish, cattle, wine or flowers which all use an auction to allocate commodities. An auction is an organized contest (cf. §??) that is cheap to set-up and enable the prompt selling of almost any item. It also has the crucial ability to extract the precious information that economic agents might possess regarding the item for sale; this explains the appearance of this chapter in the Part on asymmetric information. Modern references are Klemperer (2003) and Milgrom (2004).

The chapter is organized as follows: we first shed light upon the origin and main uses of auctions with an emphasis on the assignment of natural monopolies. We then compare the main auctions before inquiring into the optimal auction (for the seller) and efficient one (for society).

4.1 Purpose of Auctions

4.1.1 Origins

Auctions are a very useful and old exchange mechanism. There are records of auctions for slaves around 1900 BC in Assyria (Irak), for virgin brides and slaves around 500 BC in Babylon. In BC Rome, auctions were commonly used to sell real estate, slaves or goods as follows: an auctioneer sets a low starting price and waits for participants to signal a higher price;¹ they can call out openly or nod in which case it is the auctioneer who sets the new (higher) standing price.

¹The word auction derives from the latin “auctus” which means increase.
When no one dares to bid above the standing price, the bidder who made it gets the item and pays the seller his bid. As astounding as it may sound, the entire Roman empire was once sold in an auction!\(^2\)

In the XX\(^{th}\) century, governments have used auctions to sell treasury bills, foreign exchange, mineral rights, oil fields, assets of firms to be privatized, public land or properties and more recently air waves for television or telephone. In the private markets, houses, cars, agricultural production, livestock, art and antiques are commonly sold by auction. Even more recent are the internet auctions to sell used items and the business to business (B2B) procurement auctions whereby firms compete to sell or buy at bargain price their inputs or outputs.

4.1.2 The case for auctioning

Perfect Competition

An auction is the practical trade mechanism that comes closest to perfect competition. Standardized goods such as financial assets (stocks, options, derivatives), grain or minerals are traded in exchanges (trading posts, clearinghouses) where anonymous buyers meet anonymous sellers in a double auction (cf. §4.2.1).\(^3\) The remarkable property of such markets is to extract the private information of participants. Since nobody has market power in these large markets, the optimal pricing strategy is to be “price-taker”. As a consequence, individual demand equates (market) price and (personal) marginal willingness to pay; symmetrically, individual supply equates price and marginal cost. The bidding behavior of all participants therefore reveal perfectly all their economically relevant information and it is aggregated in the equilibrium price. We obtain the first welfare theorem according to which efficiency is reached in a competitive market.

---

\(^2\)The anglo-saxon literature refers to the traditional Roman auction as the English auction; we have not been able to find an explanation for this bizarre praxis avoided by the landmark article of Vickrey (1961). The real life auction closest to the open ascending model prevalent in the literature is a variant of the Roman auction used in Japanese fish markets.

\(^3\)Ask prices (supply) are ranked lowest to highest while bids (demand) are ranked highest to lowest. The demand and supply profiles so generated are then matched to determine an equilibrium price and an amount of trade.
Revelation Mechanism

It would seem that the previous mechanism fails to work properly to sell a unique item such as a painting, a house, a mineral field or a mobile phone license. Indeed, being so exceptional or unique, the item does not have a well known market value so that both sellers and buyers are unsure of how much they should ask or bid.

Organizing an auction is an answer to this problem. The owner needs to advertise the event to attract the largest possible number of participants because each potential buyer will bring his own small piece of information relative to the item for sale. The seller can then hope that during the auctioning process these information bits will be revealed by the participants through their bidding behavior (like in a competitive market). This way, the seller can sell the item to the person who values it most, thereby maximizing her revenue. Similarly, whenever an economic agent wants to buy or procure a service or a very specific item like a museum or a railroad line, he can use a procurement auction to attract many potential contractors (sellers of the service) and award the production of the item to the least demanding candidate in order to minimize spending.

Auctions vs. Regulation

There are many activities and services with natural monopoly features that force a government to intervene in order to avoid private monopoly distortions. Yet the private sector being more efficient at providing these services, the government is lead to auction an exclusive right over its jurisdiction. In theory, the government ought to allocate the license with a view to maximize efficiency i.e., look for the firm best able to provide high quality at low cost.

In the past, the allocation process was often a “beauty contest” where contenders would propose a detailed plan of activities and the government would select the one best fitting its needs. Such a scheme presents two obvious problems. Firstly, the promises included in the plan are hard to check and enforce, thus hard to believe in the first place which ultimately means that few elements can really be used to decide between offers. Secondly, this selection process is open to wasteful lobbying and corruption; it is widely believed that such methods have seriously limited entry, challenging and innovation (cf. Part ?? and
an issue better known as regulatory capture. Short sighted proponents of “economic liberalism” claim that the allocation should be random because once some lucky person gets the license, all interested firms will try to buy it back from him (the randomness impede firms from developing their lobbying).\textsuperscript{4} The ensuing competition among firms will guarantee efficiency: the license will go to the firm with the highest willingness to pay. Underlying the whole story is the absence of transaction costs i.e., the negotiation between parties is neither time nor lawyers consuming.

This politically oriented view was powerful enough in the US during the 1980s to force the FCC to organize lotteries to allocate radio and TV waves; this method lasted until 1993 and lead to a severe fragmentation and to very costly negotiations between telecommunication firms and all sort of arbitrageurs. Experience (and the comparison with Europe) has revealed the existence of large transaction costs. Looking at this particular case, it is quite obvious that the lucky arbitrageur who wins a license and the telecommunication firm willing to buy it back have private information; none of them knows clearly how much the license is worth for the other. The arbitrageur knows nothing about telecommunications and the firm ignores whether the arbitrageur already got an offer from a competitor. As we formally show at the end of this chapter (cf. §4.3.3), this asymmetry of information makes bargaining inefficient in the sense that the license is some times resold to a firm that does not value it most.

Hence there is a case for devising a wise allocation mechanism that extracts the information of the relevant participants and identifies directly the highest value for the license. Economists claim that auctions have fared quite well in this respect and we shall argue in that in sense in the theory section.

**Collusion**

Bidders to an auction have an incentive to collude (form an illicit cartel) in order to get the item at better conditions than if they were competing one against the other. In the case of auctions among private economic agents the presence or

\textsuperscript{4}Since most participants to these lotteries have limited liability, the price asked by the government ought to be nearly zero and in any case far from the true highest value; as a consequence, the government is relinquishing part of its budget in favor of a single firm or agent and will be forced to resort to distortionary taxes to make up for the difference. This constitutes a first drawback of this method.
absence of collusion is a matter of surplus distribution among buyers and sellers but when the public power is involved, reducing its surplus is akin to increasing public spending which causes inefficient distortions in the economy. Antitrust authorities therefore actively fight collusion in procurement auctions for public works; the large number of prosecutions and judgments against groups of firms is proof that collusion is widespread in auctions.

McAfee and McMillan (1992) show that whenever the members of a cartel are unable to make monetary transfers (because of anti-trust surveillance) the best they can do to manipulate an auction is to bid the same amount, a feature commonly observed in procurement auctions. If they are able to redistribute the rents of collusion among themselves, then they behave as a monopsonist in the original auction and resell the item among themselves in a private auction.

Robinson (1985) argues convincingly that collusion is easier in a second-price auction (open or sealed) than in a first-price one. Indeed, for an item of known value 50€, it is enough for the designated winner to bid a high value like 100€ and for all other colluders to bid a low 7€; this way no one will ever cheat. Indeed, with a bid like 101€, the cheater would end up paying 100€ and lose 50€ while if he bids 30€, he doesn't win the item but hurts the winner who will retaliate in subsequent auctions. Such a deception is much more difficult to play in a first-price auction because the designated winner ought to bid only slightly more than his mates, say 8€; but then anyone can outbid him at 9€ and make a profit of 41€.

### 4.2 Comparing Auctions

In our theoretical presentation, the original owner of the item for sale (she) stands also as the auctioneer who organizes and runs the auction. We shall first consider her encounter with a single potential buyer (he), before generalizing to several. A modern reference for this section is Milgrom (2004).

#### 4.2.1 Typology

*Auctions* can be classified according to their rules. A one-sided auction sees a unique seller or buyer proposing an item to several bidders. Two-sided or dou-
Auctions put buyers and sellers in contact through the auctioneer. Bids can be secret (sealed) or public (open). This last case opens the possibility of competition among bidders by alternating bids. We present the most frequent ones and indicate in parenthesis the familiar name of each.

- **Open ascending** (Roman): in the Japanese electronic version, the price starts low and goes up with the clock time. Bidders push a button to enter the auction and release it to leave (no reentry permitted) so that the winner is the last holding bidder; he pays the standing price. Examples: art auction at Christie’s and Sotheby’s (London) since the XVIIIth century or *Drouot* (Paris) since the XVIth century.

- **Open descending** (Dutch): the price starts from a very high level and falls with the clock time until someone says mine (pushes a button); the winning bidder gets the item and pays the standing price. Examples: Flower sales in the Netherlands (cf. case study by Kambil and van Heck (1996)) or the Google IPO.

- **First-price sealed** (Sealed): Bids submitted secretly in written form to the auctioneer who sorts them and award the item to the highest bidder who pays his bid. Examples: procurement contest or airwaves licenses.

- **Second-price sealed** (Vickrey (1961)): same as before except that the winner pays the second-highest bid. Example: stamps sales by mail in the US during the late XIXth.

- **All-pay**: every bidder pays his bid and the highest bidder receives the object. Examples: any lottery (sweepstakes) or equivalently any match opposing two teams; each participant’s bid is the effort he produces to win the match.

- **War of Attrition**: bidders put repetitively an equal amount (of money, time or effort) on the table until all but one drop out. The item goes to the last standing bidder.\(^6\)

---

5. If the price in a Roman auction increases by very small amounts, the last bidder ends up paying the second highest bid plus a very small amount just as in the Japanese version.

6. Attrition is a second-price format whereas the (standard) all-pay auction is a first-price format. Indeed, the highest bidder need only expand a little more than his last opponent. The terminology derives from warfare and was first theorized in biological competition.
• **Double Auction:** participants submit supply and demand bids, then the auctioneer sort demand bids in descending order and supply bids in ascending order. Matching the two curves yields an equilibrium price at which all feasible transactions are executed.

Drawing on the characteristics of the double auction, Walras (1874) offers a theoretical description of a competitive market: the auctioneer (called Walrasian to distinguish from real ones) cries out a price and waits for demands and supplies. He then lowers the price if supply exceeds demand and raises it otherwise. This “tâtonnement” process eventually converges to an equilibrium price. Today most electronic markets use an automated Walrasian auctioneer that offers an equilibrium price continuously equating demand and supply.

The other crucial element in the study of auctions which justifies its presence in this Part is the information setting, more precisely what the seller and the potential buyers know regarding the item for sale. In the case of *private values*, each bidder has a privately known willingness to pay for the item, called the value and no one else knows it. This applies for a fixed quantity of raw standard material or a license for a mobile telephone network because the willingness to pay of a potential buyer is determined by the technology where the item will be used as an input. In the *common value* situation, the item for sale has a unique market value which is all that matters for buyers but no one knows it exactly e.g., an oil field. Lastly we can *mix* the previous settings, letting the item having an intrinsic objective value but also a subjective one for each bidder e.g., a painting which you might appreciate on top of knowing that it also has a market value for resale.

Before delving into theory to compare the 4 *standard* one-sided auctions, let us recall that the obvious advantage of open cry auctions is the speed at which large quantities can be sold; this is particularly crucial for fresh food or flowers and explain why Roman and Dutch auctions have been used for centuries.

### 4.2.2 Standard Auctions

Some formal comparisons can be made among the 4 *standard* one-sided auctions. In all the sealed auctions a strategy is simply a price (to be written down the porposal) while in the open auctions, a strategy is a stopping rule, a limit
price that can depend on what the player has observed while the standing price was evolving.

First-Price Auctions: Dutch and Sealed

In the Dutch auction no information is revealed until some says mine triggering the end of the auction, thus a strategy is a single figure just like in a sealed auction. Given that in the Dutch and Sealed auctions, the winner pays his bid, these two auctions are strategically equivalent; this is true whatever the information context. Hence we might see the Dutch auction as open first price. What distinguishes them in the real life relates mostly to collusive behavior among bidders and transaction costs, the speed at which it is conducted and can be repeated.

Second-Price Auctions: Roman and Vickrey

When values are private, the Roman and the Vickrey auctions are also equivalent albeit in a weaker sense. In the Roman auction, if values are private and statistically independent then a player learns nothing from the fact that some people are dropping out as the price goes up. In fact, it is a dominant strategy (optimal whatever do the other bidders) for him to stay until the public price reaches his own value because dropping out before is to lose an opportunity to get the item cheap while staying too long is taking the risk of buying dear.

In a Vickrey auction, a strategy is a price \( \hat{v} \) (equal or different from the true value \( v \)). The optimal strategy for a bidder, say #1, is to play his personal value \( v \) whatever the behavior of other bidders and whatever the information structure. Indeed, letting \( u \) be the highest bid made by someone else, say #2, we see in Table 4.1 that telling the truth is identical or better than understating and also identical or better than overstating. The \( \star \) cells indicate when lying reduces the payoff of bidder #1. In both cases, the winner is the bidder who values most the item but he pays only the second highest valuation, thus the Roman auction could be relabeled “open second price” since the Vickrey is the sealed-bid second price auction.
Affiliated information

The private information of bidders is rarely independent, rather it is often affiliated in the sense that when one bidder is optimistic i.e., receive information stating that the item for sale is very valuable, it is more likely that other bidders’ are also optimistic. In this context and assuming risk-neutral bidders, whose signals are drawn from symmetric distributions, and whose value functions are symmetric functions of the signals, Milgrom and Weber (1982) show that the following ranking in terms of seller’s revenue: *Roman* ≫ *Vickrey* ≫ first-price auctions (*Dutch* or *Sealed*). This theory is backed by the prominent use of the *Roman* auction through history and settings.

The intuition behind this classification is that the surplus of the winning bidder is due to her private information. Hence to maximize revenue, the seller looks for an auction able to extract the winner’s information. When information is affiliated, this effect will be stronger the more the price paid depends on others’ information. The standard auction where the price most depends on all bidders’ information is the *Roman* auction; indeed, the winner has seen everybody else drop out and has been able to infer much of these events so that his own winning bid reveals a maximum amount of private information. In the *Vickrey* auction, a similar but weaker phenomenon takes place because the price depends only on the second-highest bid. Lastly, in the first-price auction (*Dutch* or *Sealed*), a player’s bid incorporates no information in addition to his own.

By the same token, if the seller has some private information, he should release it to augment the information at the disposal of bidders since this will motivate them to bid higher. The general principle stating that expected revenue is raised by linking the winner’s payment to information that is affiliated with the winner’s information, is known as the Linkage Principle. In practice, sellers of art or exploration rights pay independent experts to assess and reveal the likely value of the item for sale.

<table>
<thead>
<tr>
<th>bidder #1 / bidder #2</th>
<th>$u &lt; \tilde{v}$</th>
<th>$\tilde{v} &lt; u &lt; v$</th>
<th>$v &lt; u &lt; \tilde{v}$</th>
<th>$\tilde{v} &lt; u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>understate $\hat{v} = v &lt; v$</td>
<td>win, $v - u$</td>
<td>lose, 0 ★</td>
<td>lose, 0</td>
<td>lose, 0</td>
</tr>
<tr>
<td>truth $\hat{v} = v$</td>
<td>win, $v - u$</td>
<td>win, $v - u$</td>
<td>lose, 0</td>
<td>lose, 0</td>
</tr>
<tr>
<td>overstate $\hat{v} = \tilde{v} &gt; v$</td>
<td>win, $v - u$</td>
<td>win, $v - u$</td>
<td>win, $v - u$ ★</td>
<td>lose, 0</td>
</tr>
</tbody>
</table>

Table 4.1: Gain for bidder #1
4.3 Optimal Auctions

4.3.1 Revenue Equivalence

Vickrey (1961)’s revenue equivalence theorem states that if a fixed number of identical, risk-neutral bidders, who each want a single unit, have independent information, and bid independently, then all 4 standard auctions yield the same expected revenue to the seller.

Revelation Principle

We follow Myerson (1981) to prove this important result. The first step is to demonstrate Myerson (1979)’s revelation principle: When a person dressed in red participates in the auction (or any other selling mechanism with very complex rules) taking a series of actions, the seller can record them on a sheet and title it “red strategy”; she can do likewise with all the participants dressed in blue, green or any other color. The fact that the red dressed man plays the red strategy in equilibrium of the auction game means that, for him, it dominates the green strategy; likewise the green strategy dominates the red one for the green dressed man. Since the preferences of participants are captured by their WTP for the item, any two people with the same WTP will behave identically. Thus, the seller will record (possibly) different strategies only for people with different WTPs. This imply that we can drop the color labeling system and use instead one based on the WTP of participants.

The second step is to use this principle to characterize a participant’s payoff. A bidder with WTP \( v \) (his private information) will get the item with a probability \( \phi(v) \) that depends on the rules of the auction, the behavior of other participants and obviously his own optimal strategy.\(^7\) Likewise, he expects to pay some amount\(^8\) \( t(v) \), so that his expected surplus is \( u(v) = v\phi(v) - t(v) \). In equilibrium, it does not pay for bidder \( v \) to act as if he was \( \hat{v} \) i.e., use the optimal strategy of that person so as to win the item with probability \( \phi(\hat{v}) \) and pay an expected \( t(\hat{v}) \). Analitically, this reads: \( u(v) \geq v\phi(\hat{v}) - t(\hat{v}) = u(\hat{v}) + (v - \hat{v})\phi(\hat{v}) \). Since it won’t pay

\(^7\)We consider a Nash equilibrium of the auction game where bidders use optimal strategies.

\(^8\)The payment is not necessarily linked to being awarded the item; in an all-pay auction for instance, all bidders pay their bid.
either for $\hat{v}$ to act as if he was $v$, we obtain
\[ \varphi(\hat{v}) \geq \frac{u(\hat{v}) - u(v)}{\hat{v} - v} \geq \varphi(v) \] (4.1)
out which we deduce two results. Firstly,
\[ \hat{v} \geq v \Rightarrow (\hat{v} - v)(\varphi(\hat{v}) - \varphi(v)) \geq 0 \Rightarrow \varphi(\hat{v}) \geq \varphi(v) \] (4.2)

By playing optimally in an auction, a bidder with higher WTP guarantees himself a greater probability of winning the item i.e., $\varphi' \geq 0$.

The second result is obtained by letting $\hat{v}$ converge towards $v$; at the limit $u'(v) = \varphi(v)$ must be true. Integrating this equality, we can write
\[ u(v) = u(0) + \int_0^v \varphi(x) \, dx \] (4.3)

The interpretation goes as follows: given that the bidder follows an optimal (equilibrium) bidding strategy, his final surplus depends only on the probability of winning (and his payoff in the worst case). Notice that because $\varphi$ is increasing, $u$ is convex meaning that additional WTP becomes more and more valuable to win the item at a good price and earn a maximum surplus.

Whenever two auctions generate for all bidders the same probabilities of winning and the same surplus in the worst case then they generate the same surplus function $u(.)$, the same payment function $t(.)$ and finally the same revenue for the seller; this is the general form of the revenue equivalence theorem. In the 4 standard auctions, the item goes always to the bidder who has the highest value thus the probability of winning is the same because it depends only on the statistical distribution of values. Furthermore, a bidder drawing the lowest value $v$ cannot win (with positive probability) thus her surplus is always 0 so that the theorem applies yielding our initial claim.

**Optimal Bidding**

As we already saw, the optimal bidding strategy in the Vickrey auction with independent values is one's own value, a behavior known as truth telling or truthful revelation (of the private information). If values are private and independent
then the same is true in the Roman auction.

The revenue equivalence enables to compute the optimal bid in first-price auctions because expected payments are identical to those of second price auctions. Now, the expected payment is the product of the winning probability by the expected price, and we know that the winning probability is the same in all 4 standard auctions if values are private and independent. In that case, the bid $b_i(v)$ in the first-price auction is equal to the expected price $\mathbb{E}[\hat{v} | \hat{v} < v]$ in the second price auction with $\hat{v}$ being the second highest bid. Hence, my optimal bid in a first-price auction is the expected second highest value conditional on being lesser than my own.

Let us compute this for player #n. We first look for the distribution of the random variable $u$, the highest bid of the other $n - 1$ players. Since all values are drawn form the distribution $H$ with density $h = H'$, the probability that player #1 bids in $[\hat{v}; \hat{v} + d\hat{v}]$ is $h(\hat{v})$ while the probability that the remaining $n - 2$ players bid less than $\hat{v}$ is $H(\hat{v})^{n-2}$; since identities do not matter the density of $\hat{v}$ is $g(\hat{v}) = (n - 1) h(\hat{v}) H(\hat{v})^{n-2}$. Using integration by parts below, we obtain the optimal bidding strategy as

$$b(v) = \mathbb{E}[\hat{v} | \hat{v} < v] = \frac{\int_0^v xg(x) \, dx}{\int_0^v g(x) \, dx} = v - \int_0^v \left( \frac{H(x)}{H(v)} \right)^{n-1} \, dx \tag{4.4}$$

which, as expected, is lesser than the value to reflect the fact that players want to make a profit in these first-price auctions and are aware of the possibility to win the item without bidding their private value.

To give a practical use to formula (4.4), let us assume that each buyer’s value is uniformly distributed over $[0; \bar{v}]$ i.e., $H(v) = v / \bar{v}$, then $b(v) = v - \int_0^v \left( \frac{x}{\bar{v}} \right)^{n-1} \, dx = v - \left[ \frac{x^n}{n \bar{v}^{n-1}} \right]_0^v = v - v / n$. Notice that the optimal bid does not depend on the top value but only on the number of contenders. We see here why it is interesting to attract many participants in an auction: it motivates all of them to bid more aggressively which raises the seller’s revenue.

A phenomenon well known among practitioners is the winner’s curse according to which the winner of an auction has paid more than the value of the item. Whenever some people receive optimistic information while others receive pessimistic information, values are not private anymore, they become correlated (common value model). In that case, the winning bid, being made by the player
who received the highest signal, is overtly optimistic and frequently overshoots the true value of the item. In theory, player take this phenomenon into account when computing their optimal bid. The key to compute my optimal bid is to forget about the item's expected value (based on my private information) and concentrate on its value when my bid is the highest, hence taking into account that other did not dare bid so high.

4.3.2 Optimal Selling Mechanism

Intuition

Bulow and Roberts (1989) introduces this topic with the follow example: Wallace would be ready to pay some price between 0 and 10 while Gromit would pay some price between 10 and 30 for a painting. In a Roman auction, Gromit systematically outbids Wallace and pays exactly his value so that on average the seller gains 5. Setting a starting price of 10 would raise that payoff to 10 because Wallace would never bid and Gromit would wisely bid his minimum WTP. An even better idea is to set a starting price of 15. True, not even Gromit would participate when his value is less than 15 (probability $\frac{1}{4}$) but the average revenue would jump to $\frac{3}{4} \times 15 = 11.25$. The question is then to identify criteria that can help us build rules of an optimal auction.

Observing that the seller's pricing problem is quite similar to that of a standard monopolist, one can assimilate the distribution of bidders' values to a demand curve and compute a marginal revenue curve. An optimal (revenue maximizing) auction is then one where the item goes to the bidder whose marginal revenue is greatest; this includes the seller herself if no marginal revenue surpass her reservation value $v_0$. This scheme is implemented by a modified second-price auction where each participant bids a value (openly or secretly), then the seller then computes the marginal revenues and gives the item to the bidder with the greatest marginal revenue. This auction is incentive compatible in the sense that participants truthfully reveal their value.

The previous analysis of an optimal auction (for the seller) makes the search for an efficient auction (socially optimal) quite simple. We only need to give the

---

9 Check that with a starting price $s$, the expected revenue is $\frac{30-s}{30-10} s$ and is maximum for $s = 15$. 

91
item to the bidder with the highest valuation; this includes the seller herself if no bidder value surpass $v_0$.

To achieve this, we can use the Vickrey auction with reserve price $v_0$ since we already proved that bidders reveal their true WTP in that auction. By the revenue equivalence theorem, the 4 standard auctions with reserve price $v_0$ are efficient. This result looks as a promising allocation mechanism but a word of caution is necessary when dealing with the auctioning of governmental licenses. As we explain in §??, an incumbent operator (already owning a license) has more to lose from entry than the challenger can hope to win after entry, thus the incumbent will strategically raise his bid for a second license to outbid the challenger in order to block his entry. This could lead to an inefficient outcome if the challenger had a better technology or simply because a monopoly is maintained, instead of evolving towards a more competitive duopoly market structure.

To compare the previous optimal selling mechanism and the efficient one, we observe that since the marginal revenues $\tilde{R}_{m,i}$ of the potential buyers are all increasing, the ranking of values yields the same ranking of marginal revenues so that the object goes to the same person whenever it is sold. If it was not for the monopolist reserve price $p^M$ which is greater than his true valuation $v_0$, the optimal selling mechanism would be efficient.

**Discriminating Monopoly Analogy**

We now develop the previous intuition to uncover the optimal selling mechanism. If the seller was an omniscient druidess, she would be able to rank the WTP of participants $v_1 \geq v_2 \geq ... \geq v_n$ and ask bidder #1 to pay $v_1$ in a “take-or-leave-it” manner i.e., act like a perfectly discriminating monopoly (cf. §??), rip a maximum revenue and perform an efficiency enhancing trade since the item would go to the economic agent best able to use it (unless her own valuation $v_0$ is greater so that efficiency commands her to keep the object). In real life situations, the WTP of participants is a private information to each of them. Maximizing revenue is therefore akin to maximizing information revelation. We adopt Bulow and Roberts (1989)’s heuristic analogy of auctions with monopoly price discrimination.

The seller’s dilemma is almost identical to that of a monopoly facing a market demand because facing one buyer whose WTP you ignore is formally identi-
cal to facing a large population of indistinguishable buyers. Indeed, a typical demand curve $D(.)$ like that displayed on Figure ?? can be seen as a series of $n = D(0)$ people ready to buy a single unit of the good, ranked by their WTP. At the price $p$, monopoly sales are the fraction $D(p)/D(0)$ of the market size i.e., there is a proportion $H(p) \equiv 1 - D(p)/D(0)$ of buyers whose WTP is lesser than $p$. An alternative way to read this is the following: if a potential buyer is picked at random from the population and offered the item for the price $p$, he will accept with probability $D(p)/D(0) = 1 - H(p)$.

Now, an auctioneer does not pick a buyer, rather a buyer presents himself at the auction and although she ignores how much he would be ready to pay for the item, she knows his population of origin. The latter is completely described by the distribution function $H(.)$ which we assume known to the auctioneer, just like we always assume that a monopolist knows the demand function $D(.)$.

### Marginal Revenue

Consider the encounter between the seller of the item and a potential buyer drawn from a population whose statistical distribution is $H$. When the seller offers the item for the price $p$, the probability of a greater buyer WTP is $1 - H(p)$ which is also the probability of a sale; this amount plays the role of quantity $q$ in the standard monopoly analysis as illustrated on the left panel of Figure 4.1. From an ex-ante point of view, the expected revenue is $\bar{R}(p) \equiv p \times (1 - H(p))$ which can be written $R(q) = qH^{-1}(1 - q)$ using the $p \rightarrow q$ change of variable. We can then compute the marginal revenue $R_m$ and using once again the change of variable $q$ into $p$, we derive

$$\bar{R}_m(p) = R_m(d(p)) = p - \frac{1 - H(p)}{h(p)} < p$$

which is plotted\(^\text{11}\) on the right panel of Figure 4.1 for the uniform distribution over $[0; \bar{v}]$. Using the change of variable $p = H^{-1}(1 - q)$ in $R(q) = \int_0^q R_m(x) \, dx$,

\(^9\)A monopoly sells many units while the auctioneer has typically a single item for sale.

\(^1\)For most statistical distributions, the hazard rate $\frac{h(.)}{1 - H(.)}$ is increasing so that $\bar{R}_m(.)$ is also increasing. Furthermore, we have constructed $H$ from a regular demand function admitting a decreasing marginal revenue.
one obtains:

\[ p \left( 1 - H(p) \right) = \tilde{R}(p) = \int_p^{\tilde{v}} \tilde{R}_m(v) \, dH(v) \Rightarrow p = \mathbb{E}[\tilde{R}_m(v) | v > p] \]  

(4.6)

meaning that the price is the expected marginal revenue conditional on the bidder’s value being larger than this price.

![Figure 4.1: Virtual Utility](image)

**Optimal Bilateral Sale**

We devise here the optimal mechanism to sell one item to someone whose WTP is drawn form the distribution \( H \). The seller’s opportunity cost is \( v_0 \). The expected surplus of the potential buyer, \( \mathbb{E}[u] \), is computed from (4.3):

\[
\mathbb{E}[u] = \int_0^{\tilde{v}} u(v) \, dH(v) = u(0) + \int_0^{\tilde{v}} h(v) \, dv \int_0^v \varphi(x) \, dx \\
= u(0) + \int_0^{\tilde{v}} \varphi(x) \, dx \int_0^v h(v) \, dv = u(0) + \int_0^{\tilde{v}} \varphi(v) (1 - H(v)) \, dv \\
= u(0) + \int_0^{\tilde{v}} \varphi(v) \frac{1 - H(v)}{h(v)} \, dH(v) 
\]

(4.7)

by an exchange of integration order.
A bidder with WTP $v$ is willing to take part in the auction if $u(v) \geq 0$ and this constraint will be always satisfied if it is satisfied for the lowest possible WTP i.e., the only participation constraint that matters is $u(0) \geq 0$. Since revenue is $t(v) = v\varphi(v) - u(v)$, we can use (4.7) to check that the seller earns on expectation:

$$
E[t] = \int_0^{\tilde{v}} \left( v - \frac{1 - H(v)}{h(v)} \right) \varphi(v) \, dH(v) - u(0) = \mathbb{E}[\varphi \tilde{R}_m] - u(0) \quad (4.8)
$$

Taking into account her opportunity cost $v_0$ of renouncing to the item, her (producer) surplus\(^{12}\) is

$$
W_S = \mathbb{E}[\varphi \tilde{R}_m + (1 - \varphi) v_0] - u(0) = v_0 - u(0) + \mathbb{E}[\varphi (\tilde{R}_m - v_0)] \quad (4.9)
$$

so that its maximization leads her to sell the item i.e., set $\varphi(v) = 1$ only for values $v$ such that $\tilde{R}_m(v) \geq v_0$. Letting $q^M$ solve $R_m(q) = v_0$ and $p^M$ solve $1 - H(p) = q^M$, we check on the left panel on Figure 4.1 that this optimal rule generates sales to a proportion $q^M$ of potential buyers which is exactly the standard monopoly quantity given that the opportunity cost $v_0$ is the marginal cost of “producing” the item. Furthermore, the rule satisfies the condition $\varphi' \geq 0$ since $\varphi$ is nil over $[0; p^M]$ and unitary over $[p^M; \tilde{v}]$.

To implement this outcome it is enough to behave as a standard monopoly i.e., ask a price $p^M = H^{-1}(1 - q^M)$ for the item or to set-up the following auction: the buyer bids a WTP $\hat{v}$ and is allowed to buy the item at the price $p^M$ if the marginal revenue corresponding to his offer, $\tilde{R}_m(\hat{v})$, is greater than $v_0$. It is a simple exercise using the left panel of Figure 4.1 to check that a buyer with WTP $v$ has no interest to lie i.e., he will truthfully reveal $\hat{v} = v$ (the proof for the general case is provided in the next paragraph).

When comparing the two previous mechanisms, the second looks dumb since it adds a bidding stage that seems irrelevant. That is correct in the present setting but once there are several bidders, it becomes a useful device to force a maximum revelation of information.

\(^{12}\)Recall that profit is $\Pi = W_S - v_0$. 
Consider \( n \) independent but not necessarily identical bidders i.e., bidder \( #i \)'s value \( v_i \) has statistical distribution is \( H_i \) with density \( h_i \). We count the seller as a dummy bidder \( #0 \) whose value distribution is entirely concentrated at \( v_0 \) and set \( t_0 = u_0 = 0 \).

When participating in the auction, each bidder will bid optimally so that the previous results will apply. We denote \( \varphi_i(\mathbf{v}) \) the probability that bidder \( #i \) wins the object when the vector of values is \( \mathbf{v} \). Thanks to the dummy bidder trick, the seller’s revenue is also her producer surplus

\[
W_S = \mathbb{E} \left[ \sum_{i \geq 0} \varphi_i(\mathbf{v}) \tilde{R}_{m,i}(v_i) \right] - \sum_{i \geq 0} u_i(0) \tag{4.10}
\]

where \( \tilde{R}_{m,i} \) is computed as in (4.5) but using the distribution function \( H_i \).

An optimal (revenue maximizing) auction is now easy to identify: for every vector \( \mathbf{v} \), the item should go to the bidder whose marginal revenue \( \tilde{R}_{m,i}(v_i) \) is greatest; this includes the seller herself if no marginal revenue surpass \( v_0 \).\(^{13}\) It remains to build an auction where this outcome is implemented for each possible combination of private values.

Consider the \textit{modified second-price auction} where each participant bids a value \( v_i \) (openly or secretly), the seller then computes the marginal revenues \( r_i \equiv \tilde{R}_{m,i}(v_i) \) for \( i \geq 1 \) and \( r_0 \equiv v_0 \). Assume for simplicity that after ranking these marginal revenues we have \( r_1 \geq r_2 \geq \ldots \geq r_n \); let then \( q_2 \) be defined by \( \tilde{R}_{m,1}(r_2) = q_2 \). The object is awarded to bidder \#1 at the price \( p_2 = H_1^{-1}(1 - q_2) \) as shown on Figure 4.2. Notice that since the price is always positive, a bidder with minimum WTP never wins it, thus derives a nil utility so that \( u_i(0) = 0 \).

We claim that everybody has an incentive to announce truthfully his value in this auction. There are two cases to consider; either you are a winner, say bidder \#1 with the truthful bid \( v_1 \) or a strategic bid. In the first case, the price paid is a function of \( r_2 \) not \( r_1 \). As can be seen on Figure 4.2, overstating \( v_1 \) by announcing \( \hat{v}_1 > v_1 \) has no effect whatsoever. Understating \( v_1 \) by announcing \( \hat{v}_1 < v_1 \) will have no effect either meanwhile \( \hat{v}_1 > p_2 \) but if the understatement is too strong,

\(^{13}\) Since we assumed \( \tilde{R}_{m,i} \) increasing, a higher value \( v_i \) leads to a higher marginal revenue, thus a higher or equal probability of winning; this means that the necessary condition (4.2) \( \varphi' \geq 0 \) is satisfied.
the item will go to someone else since the computed marginal revenue \( \hat{r}_1 \) is now inferior to \( r_2 \); this represents a loss since telling the truth leaves a net surplus \( v_1 - p_2 > 0 \). Now, for a bidder, say \#3, who does not win when truthfully bidding \( v_3 \), there is nothing to do. Indeed, the only way to win the object is to outbid the current winner (\#1) with some bid \( \hat{v}_3 \) that has to be greater than \( p_3 \) the price computed using \( H_3 \) and the winning marginal revenue \( r_1 \) which itself is larger than the true valuation \( v_3 \) as can be seen on the right panel of Figure 4.2.\(^{14}\)

When the auctioneer has absolutely no discriminating information regarding the participants, the distribution functions are identical. This implies that the highest value translate into the highest marginal revenue, thus our optimal auction becomes the Vickrey auction with an optimal reserve price. By the revenue equivalence theorem, the 4 standard auctions are optimal. This result is correct for private as well as common value settings.

**Efficient Auction**

An efficient auction is one that maximizes welfare

\[
W = \mathbb{E} \left[ \sum_{i \geq 0} v_i \phi_i(v) \right]
\]

\(^{14}\)Notice that the reasoning we’ve used here was identical to that shown in Table 4.1.
thus, it is enough to award the object to the highest WTP, including the seller as a dummy bidder with value $v_0$.

4.3.3 Bilateral Trade under Uncertainty

Informed Trade

When two people are interested into trading an item, it is enough that the value $b$ for the potential buyer be larger than the value $s$ to the current owner (and potential seller); they might agree on the price $\frac{b+s}{2}$ if they have equal bargaining abilities (cf. §??).

This simple analysis underlies the case for “free trade” or (economic) “liberalism”: whenever something in the economy is owned by someone, say a man, unable to make good use of it then someone else, say a woman, will offer to buy the item. The rationale behind this offer is that she thinks she’s able to create more market value from the item, hence she is ready to pledge enough money to buy it because later on she will be able to recoup the investment. The immediate conclusion is that initial ownership does not matter, only ability does.\(^{15}\)

This story runs into a difficulty as soon as the valuations $b$ and $s$ are not public knowledge. If each agent only knows his or her own valuation then bargaining over the exchange price is more difficult and can sometimes fail to implement the efficient outcome i.e., the item might be exchanged when it should not be and conversely, it might fail to be traded when although it would have been desirable.

Information Revelation

The Vickrey (1961)-Clarke (1971)-Groves (1973) mechanism (VCG) provides a solution to this problem and more general ones; it involves a broker and the two agents. Each agent announces his value, $\hat{b}$ for the buyer and $\hat{s}$ for the seller. Trade occurs if $\hat{b} > \hat{s}$ in which case the buyer has to pay $\hat{s}$ to the broker who pays $\hat{b}$ to the seller. Exactly as in the case shown in Table 4.1, no one has an interest

\(^{15}\)The public policy implication is that governments should only aim at providing education to those who cannot pay for it (on top of its regalian missions). This way, the most brilliant and hard-working people will successfully bid for the control the scarce economic resources, generate the highest added value and in the mean time grab for themselves a fair reward.
to lie because the payment does not depend on one’s own announcement and lying can only bring inefficiency in trade. The item is therefore traded exactly when it is efficient to do so (iff \( b > s \)); yet the broker exactly loses \( b - s \), the gain from trade. Hence, efficiency can be reached but it requires some prior funding by the parties to make the broker willing to play his part.

To see if there is a way around this deficit issue, we assume the individual values \( b \) and \( s \) are drawn from a continuous distributions \( F_b \) and \( F_s \) with support \([v; \bar{v}]\) for \( v = b, s \). Both parties know the statistical information regarding their partner’ value but do not observe its actual realization. If \( b \geq \bar{s} \) then it is always efficient to sell the item and partners know perfectly this fact: although the buyer ignores the exact value \( v \), he knows that \( b > v \) for sure and likewise, the seller who ignores the true \( b \) knows that \( b > v \). Somehow, the asymmetry of information does not matter; a balanced and efficient mechanism is a sales contract stipulating any fixed price \( p \in [\bar{s}; b] \). Since \( b \geq p \) and \( p \geq s \) for sure, it is always in the interest of both the seller and the buyer to sign this contract, whatever their own valuation and without worrying for the actual value of the partner. The price will be negotiated ex-ante using the expected values of \( b \) and \( s \); for instance, the fair division price is \( p = \frac{E[s+b]}{2} \).

### Problematic Trade

The more realistic but more problematic case is that where ownership swapping is not always the efficient decision (\( b < \bar{s} \)). Chatterjee and Samuelson (1982) (CS) consider the following simple exchange mechanism whereby the seller asks a price \( p_s \), the buyer offers a price \( p_b \) and exchange takes place for the price \( \frac{p_s + p_b}{2} \) if and only if \( p_s \leq p_b \).

These authors show that the optimal strategies are to announce the non-truthful prices \( p_b^* (b) = \frac{2}{3} b + \frac{\bar{s} + 3b}{12} \) and \( p_s^* (s) = \frac{2}{3} s + \frac{3\bar{s} + b}{12} \). In equilibrium, trade occurs at price \( \frac{p_s + p_b}{2} = \frac{2(s+b)+\bar{s}+b}{6} \), if and only if \( p_b \leq p_s \Leftrightarrow b - s \geq \frac{\bar{s} - b}{4} \) which is inefficiently rare as soon as \( b < \bar{s} \); the fact that trade might be inefficient (socially undesirable) distorts the optimal bidding strategies away from truthful revelation which is the only way to be always efficient in trade.

Let us now analyze the optimal bidding strategies. The seller can ask a very high price and make sure he keeps the item, thus he can guarantee himself his private valuation \( s \). One would think that asking more than the valuation
is an even better strategy, since trade takes place only if \( p_b \) is larger so that the actual price \( \frac{p_s + p_b}{2} \) is mechanically greater than the valuation. The optimal strategy does not follow this too simple intuition; indeed, \( b < \bar{s} \), which is true, implies \( p_s^*(\bar{s}) < \bar{s} \) i.e., the seller strangely understates his valuation when the latter is very large. The reason is that trade will actually take place only if \( b > \bar{s} + \frac{\bar{s} - b}{4} \) and in that case the price will be greater than \( \frac{11\bar{s} + b}{12} \) i.e., for some unfrequent values of \( b \), the seller will make a loss but this will be more than compensated by the increased frequency of profitable trades. The same observations apply in a completely symmetric manner to the buyer. The crux of the problem here is whether trade is efficient or not is to be discovered by the parties, hence

The optimal strategies must not only concentrate on making a gain from trade but also on making trade happen.

**Limiting Efficiency Losses**

Generalizing this study, Myerson and Satterthwaite (1983) (MS) show that a bilateral trading mechanism cannot be at the same time efficient in trade, balanced in budget and guarantee the participation of both agents i.e., there is no way to eliminate the deficit issue in the VCG mechanism without introducing some inefficiency in trade. The intuition uses our previous results. Suppose the broker tries to devise a trading mechanism that is always efficient and the least costly to him. The efficiency will be satisfied only if both the seller and the buyer reveal their private information to the broker. The latter then tries to buy cheap on one side and sell dear on the other. Provided that the broker has learned \( s \), the buyer should get the item as soon as his own value is greater. As a consequence, the price paid is exactly \( s \) for otherwise some socially beneficial trade opportunities would be lost. In that case, the buyer receives all gains from trade. By an exact symmetry, the seller will also receive all gains from trade from the broker. The latter can do not better than loose the expected gains from trade.

MS further demonstrate that the CS trade mechanism maximizes the gains of trade among budget-balanced mechanism guaranteeing participation of both agents. The previous result was akin to show that a natural monopoly forced to price at marginal cost would incur losses while this new result is akin to show
that average cost pricing is the least damaging deviation from efficiency that avoids losses. As noticed by Bulow and Roberts (1989), the latter is a Ramsey-Boiteux problem.

To fix ideas, assume $b = s = 0$, $\bar{b} = \bar{s} = 1$ and that values are uniformly distributed. In the first-best world (efficient trade with a sponsor broker), the gains of trade are $\int_0^1 \int_0^b (b - s) \, ds \, db = \frac{1}{6}$. In the second best world where trade occurs only if $b - s > 1/4$, the gains of trade are only $\int_0^1 \int_0^{b-1/4} (b - s) \, ds \, db = \frac{13}{96}$, a loss of $\frac{\frac{1}{6} - \frac{13}{96}}{\frac{1}{6}} \approx 19\%$. 
Chapter 5

Entrepreneurship

According to Schumpeter (1942), *capitalism is a dynamic evolutionary process coming from within the economic system. It does not develop by adapting to exogenous changes but by mutating in a discontinuous fashion, succumbing to revolutions which displace old equilibria and structures to create radically new ones. This process of creative destruction is the essence of capitalism.*

The key actor in this enduring vision is the *entrepreneur* whose characteristic trait is *innovation*, the ability to do new things or perform old ones differently. Put differently, he/she “does the right thing” whereas a (good) *manager* “does things right”.¹ This novelty often stems from invention or research but not necessarily as its driving force is economic profit, not intellectual satisfaction. The entrepreneur needs control over the means of production to “get things done”; in that respect, ownership is helpful but hardly necessary. The innovation can be the introduction of a new item (good or service), a new technology to produce an old item, a new commercial strategy (new market, new source of supply), a new internal organization or a new market structure such as monopolization.

The reason why entrepreneurship is so important for government policy is because technical change and innovation explains much of the steady growth in advanced economies since the industrial revolution.² Most of the academic interest towards entrepreneurship is found in the finance and business literature although many important contributions use the standard toolbox of information economics.

¹As wittily stated by Marshall (1907) in a related matter, “the government could print a good edition of Shakespeare’s works, but it could not get them written”.

²Capital accumulation and the expansion of the labor force matter also but are less decisive.
In this chapter, we propose a self encompassing introduction to moral hazard and adverse selection within a single framework, the bilateral agency relationship between an entrepreneur and an investor. The former is innovative but penniless and is thus forced to seek external finance to raise the necessary capital to start up his/her project. The first section will look at the inefficiencies created by equity finance while the next one concentrates on debt. Our findings apply word for word to just about any firm insofar as managers can be assumed to perfectly represent the owners. The last section is thus devoted to managerial incentives.

5.1 Agency Cost of Equity Finance

In this section, we show that outside equity can be a source of moral hazard and adverse selection. The first segment sets the entrepreneur’s playing field and relates the financial perspective to the economics one. We then enter the heart of the matter to show the sharing of future profits through the emission of equity demotivates the entrepreneur and rationally lead him to under-invest. In the next segment, we explain why going to the equity market always conveys bad news about the quality of the firm. As a consequence, less money is raised and more generally, good firms are underpriced; this is an instance of adverse selection. Lastly, we study how the retention of a large fraction of the equity by a risk averse entrepreneur may act as a signal of quality to counter the previous “bad news” effect.

5.1.1 Background

Finance vs. Economics

The neoclassical theory of the firm used in microeconomics focuses on decisions directly relevant to market competition and tend to treat profit as a function of production and/or price. The emphasis is thus more on revenue than cost; there is also a neglect of the fact that costs are disbursed before revenues accrue, thereby creating a need for liquidity. Corporate finance, as its name indicates, aims at solving this later problem for firms (corporations) while keeping in mind that the ultimate objective remains the maximization of profits. This
amounts to invert priorities and focus on expenditure (aka. investments), treating revenue (aka. cash-flow) as a consequence. Corporate finance therefore studies what projects to undertake and how to finance them. In this paragraph we show that those views are dual or the two facets of the same objective, profit maximization.

From a financial point of view, a firm’s technology is an ability to implement projects i.e., perform investments into assets and later receive returns. We therefore identify a firm with a list of projects opportunities \((c_j, r_j)_{j \leq n}\) where \(c_j\) is the investment cost of project \(j\) and \(r_j\) its rate of return (i.e., the return is \(c_j(1+r_j)\)). As illustrated on Figure 5.1, one can rank the potential projects by decreasing rate of return. The total cost and total revenue of the first \(i\) best projects are respectively \(k_i \equiv \sum_{j \leq i} c_j\) and \(R_i \equiv \sum_{j \leq i} c_j(1 + r_j)\) which satisfy \(1 + r_i = \frac{R_i - R_{i-1}}{k_i - k_{i-1}}\).

![Figure 5.1: Investment Projects and Profitability](image)

If we pass to the continuum by considering many small size projects, then \(R\) becomes a function of \(k\) and this revenue–cost relation \(R(k)\) has derivative \(1 + r(k)\). If the risk-free interest rate is \(r_0\) then the profit function expressed in

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3 Capital is of the circulating kind i.e., it completely depreciate in the process of value creation.
net present value\footnote{If we choose to express profit in terminal value then the revenue is simply \( R(k) \) but the economic cost is then \((1 + r_0)k\) to account for the opportunity cost of time.} is
\[
\pi(k) = \frac{R(k)}{1 + r_0} - k \quad (5.1)
\]
so that the optimal level of investment \( k^* \) maximizing profit solves \( r(k) = r_0 \).

A comparative static exercise, useful later on, is to find out how \( k^* \) adjusts to changes in \( r_0 \). If the risk-free rate is initially \( r_4 \) (cf. Fig. 5.1), then the optimal investment is \( k_4 \). If the risk-free rate drops to \( r_5 \), the optimal investment increases up to \( k_5 \) while if the rate goes up to \( r_3 \) then investment recesses down to \( k_3 \).

It is now easy to relate the financial vision of profits to the economic one. Denoting \( p \) the market price of the commodity produced by the firm, the ratio of the net present value (NPV) of revenue to price \( q \equiv \frac{R(k)}{p(1+r_0)} \) can be interpreted as production. Inverting this relation yields the neoclassical cost function \( k = C(q) \) so that the profit is now
\[
\pi(q) = pq - C(q) \quad (5.2)
\]
The optimal production is that solving \( p = C_m(q) = \frac{p(1+r_0)}{R_m(k)} = \frac{p(1+r_0)}{1+r(k)} \) which, not surprisingly, leads to solve \( r(k) = r_0 \). Since \( r(k) \) is a decreasing function of \( k \), it is everywhere lesser than the average return, thus at the optimum, the firm’s average return is strictly greater than the risk-free one \( r_0 \) which means that the firm is making extraordinary profits. Such a situation characterizes a short-term equilibrium since in the long run, free entry enables competitors to claim some of these “high returns” projects and therefore reduces the overall profitability of the firm.

**Efficient Finance**

We study the contractual relationship of an entrepreneur (she) and an investor (he). The former has a limited personal wealth and owns an unalienable human capital summarized by a cash generating technology as discussed above. By its very nature, the entrepreneur’s knowledge cannot be sold so that the investment decision can only be taken by her.

To simplify notations, we denote \( R \) the present value of future cash-flow so that the project NPV is simply \( \pi = R(k) - k \) (as if the risk-free rate \( r_0 \) was nil in...
As explained above, the technology has decreasing returns to scale \( (R_m > 0, \searrow) \) so that the efficient investment maximizing the NPV is \( k^* \) solving the first order condition \( R_m = 1 \) i.e., one should equate the productive value of 1\( \varepsilon \) to its opportunity cost evaluated at market conditions.\(^5\)

If the entrepreneur was rich enough, she would be able to afford the efficient investment out of her initial wealth and efficiency would be reached (one speaks of a “first-best” situation). Nonetheless, the realistic case is precisely the opposite one and to simplify further we simply assume a zero initial wealth. The two basic ways to raise capital are *debt* where you promise repayment to a lender with an interest or emission of new *equity* where you agree to share future profits with someone else. The corresponding financial instruments are bonds and shares. The fundamental difference between them is the seniority of debt over equity in case of bankruptcy (inability to meet financial obligations).

### 5.1.2 Incentives to Under-invest

Jensen and Meckling (1976) show that recurring to *outside equity* has an agency cost because it leaves the entrepreneur with partial residual claimancy while the cost of putting time, effort or personal wealth into the firm remains the same. Thus, she is left with partial incentives toward investment. Indeed, if at the margin, 10\( \varepsilon \) invested by the entrepreneur into the firm yields in return 15\( \varepsilon \) then it ought to be invested; now if the entrepreneur gets only 50\% of it because she sold half of the firm to an outsider, she shall rationally forgo this investment. In a nutshell, “there is no way to make more than one person the residual claimant of an economic activity”; for that reason, incentives toward effort or investment are diminished for most members and under-investment occurs.

To prove this result, we use Figure 5.2 where the NPV of the project is plotted as a function of total investment \( k \). We denote \( k_0 = k^* \) the efficient investment and \( \pi_0 \equiv R(k_0) - k_0 \) the maximum NPV. To finance her project the entrepreneur sells \( \alpha \% \) of the project’s future cash-flow to an investor who in return pledges an amount \( F \). Even if these shares have voting power, the technology is uniquely controlled by the entrepreneur so that she remains the only one able to decide

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\(^5\)We also assume \( R_m(0) > 1 \) to guarantee that \( k^* \) is positive, thereby avoiding trivialities.
on the investment level. Her profit\(^6\) is \(u_\alpha(k) \equiv (1 - \alpha)R(k) - k + F\) for any investment satisfying the financing constraint \(k \leq F\). Neglecting this constraint for the moment (as if \(F\) was large), profit maximization leads her to solve \(R_m = \frac{1}{1-\alpha} > 1\). As a result, she under-invests at a level \(k_\alpha \leq k_0\) (recall the comparative static exercise of Figure 5.1). As can be grasped on Figure 5.2, the final NPV of the project, \(\pi_\alpha \equiv R(k_\alpha) - k_\alpha\) is lesser than the maximum \(\pi_0\) obtained with the efficient investment and furthermore, the larger the share sold to the outsider, the worse the resulting inefficiency.\(^7\) The intuition has already been given: since the entrepreneur has to share the profits of her firm with the investor, the marginal benefit from investing her personal wealth is lowered to \((1 - \alpha)R_m\) while the opportunity cost of money remains 1, thus the entrepreneur is demotivated and lead to under-invest. This distortion can be called an agency cost because the entrepreneur serves as an agent for the outside investor.

![Figure 5.2: Equity Financing](image)

To close the study we only need to take care of the investor; he pledges \(F\) only if \(\alpha\) is large enough but this leads to severe under-investment and a serious reduction of the cash flow generated by the project, so much that it might not cover the initial funding. Formally, the participation constraint for the investor

\[\frac{\partial k_\alpha}{\partial \alpha} = \frac{1}{(1-\alpha)^2 R''} < 0\] and \(\frac{\partial \pi_\alpha}{\partial \alpha} = \frac{\partial k_\alpha}{\partial \alpha} (R_m - 1) < 0\) since \(R_m > 1\) over the range \(k < k_0\) i.e., i.e., investment and agency value decrease with \(\alpha\) the outsider participation.

\(^6\)It is customary in the literature to use the letter “\(u\)” as in utility to denote the objective function of the agent.

\(^7\)From \(R_m(k_\alpha) = \frac{1}{1-\alpha}\), we deduce \(\frac{\partial k_\alpha}{\partial \alpha} = \frac{1}{(1-\alpha)^2 R''} < 0\) and \(\frac{\partial \pi_\alpha}{\partial \alpha} = \frac{\partial k_\alpha}{\partial \alpha} (R_m - 1) < 0\) since \(R_m > 1\) over the range \(k < k_0\) i.e., i.e., investment and agency value decrease with \(\alpha\) the outsider participation.
is $F \leq \alpha R(k_\alpha)$; it involves not just any value of $k$ but $k_\alpha$ because the investor anticipates the future choice that will be undertaken upon his acceptance of $\alpha$ shares. Combining this condition with the financing constraint $k_\alpha \leq F$, we obtain a necessary condition

$$k_\alpha \leq \alpha R(k_\alpha) \iff \alpha \geq \varphi(\alpha) \equiv \frac{k_\alpha}{R(k_\alpha)}$$

i.e., the offered share must be large enough. This constraint is obviously violated at $\alpha = 0$ since the entrepreneur is seeking funds but it is satisfied for $\alpha = 1$ because a minimal investment is still worthwhile.\(^8\) Observing now that the ratio $\varphi$ is decreasing\(^9\) with $\alpha$, the constraint is satisfied with equality for some $0 < \bar{\alpha} < 1$: this is the minimal share the entrepreneur must relinquish to get the capital necessary to undertake the investment that will then be optimal given the shares she kept. Yet, she won't offer a greater portion because she realizes that the cake to be shared with the investor, the NPV $\pi_\alpha$, is decreasing with $\alpha$ and there is no way for her to increase her absolute share of it.\(^{10}\)

Notice that in equilibrium, investors just break even, thus it is the entrepreneur who bears all the (agency) cost of his own inefficient behavior due to her inability to commit today to perform in a given way tomorrow. If she could credibly commit to the efficient investment $k_0$, investors knowing they will get a share of $R(k_0)$ would accept to pay $F = \alpha R(k_0)$ for $\alpha\%$ of the business. It would then remain to choose $\alpha = \frac{k_0}{R(k_0)}$ to satisfy the financing constraint. The crux of the problem lies obviously in the credibility of the announcement “I will invest $k_0$”, especially if it incorporates subjective elements such as human capital (the part of her working time really devoted to make the project a success).

### 5.1.3 Equity Underpricing

Myers and Majluf (1984) deal with the known difficulty of firms to raise capital

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\(^8\)Since we assumed $1 < R_m(0) = \lim_{\epsilon \to 0} \frac{R(\epsilon)}{\epsilon}$, it must be true that $1 > \lim_{\alpha \to 1} \frac{k_\alpha}{R(k_\alpha)}$.

\(^9\)Indeed, $\frac{\partial \varphi}{\partial \alpha} = \frac{\partial k_\alpha}{\partial \alpha} \frac{R - k R_m}{R^2} = \frac{\partial k_\alpha}{\partial \alpha} \frac{(1-\alpha)R - k}{(1-\alpha)R^2}$ since $R_m = \frac{1}{1-\alpha}$ at $k_\alpha$, now $k_\alpha$ being the maximizer of $(1 - \alpha)R - k$, the latter expression is positive at $k_\alpha$, hence $\frac{\partial \varphi}{\partial \alpha} < 0$.

\(^{10}\)At best for her, the participation constraint of the investor is binding i.e., $F = \alpha R(k_\alpha)$ so that her final profit is at best $\pi_\alpha - F = (1 - \alpha)R(k_\alpha) - k_\alpha$ whose derivative is $-R(k_\alpha) < 0$ by the envelope theorem ($k_\alpha$ is an optimum). Hence her payoff is maximum at $\bar{\alpha}$.
when they have private information about their current profitability. Formally, this is a problem of adverse selection similar to Akerlof (1970)’s lemons: if a firm issues equity to raise capital for an investment, it accepts to share future cash-flow with the new shareholders, a cash flow that comes from the new investment but also from the current assets. If the latter are very valuable, they will yield a lot of cash, so it not a good idea to share with foreigners; it is better to fund the new project with debt and emit zero additional equity. Potential investors understanding this logic, rightly suspect that an equity issue is a rather bad signal regarding the value of current assets; if the latter are not worth much, investors won’t pay much for the firm’s new equity either. If, contrary to this belief, the firm has really valuable assets, its equity will be underpriced, a phenomenon frequently observed in financial markets.

To see formally this phenomenon suppose that a firm owns some capital assets that will yield tomorrow a cash-flow $\tilde{x} > 0$ that is random from the point of view of investors; that is to say, the market faces a large population of firms whose future cash-flow $\tilde{x}$ has a statistical distribution $H$. We study the consequence on the quotation of the current shares of going to the equity market to finance a new project whose lumpy cost is $k$ and whose positive return rate is $r$.$^{11}$

The current true value of the firm is $\tilde{x}$, the cash-flow of current assets while its current market valuation is $\mathbb{E}[\tilde{x}]$, the expectation made by investors according to the distribution $H$. If the firm renounces to the investment when it was publicly known it had the opportunity to undertake it (event no), the value of the firm remains $\tilde{x}$ but the market value of equity becomes $V_{no} = \mathbb{E}[\tilde{x}|\text{no}]$ because investors update their beliefs given the information just revealed by the “not going” decision. Likewise, if the investment is announced together with the emission of new equity (event go), the market value of actual equity changes to $V_{go} = \mathbb{E}[\tilde{x}|\text{go}] + rk$. The true value of the firm is now $\tilde{x} + (1 + r)k$ but the value of old equity is only a fraction $\frac{V_{go}}{V_{go} + k}$ of it since it was diluted by the new emission that raised the amount $k$ needed to undertake the new project.

Comparing the two decisions and their payoffs, we see that the original owners decide to launch the new project only if the value they will retain tomorrow

$^{11}$What really matters is that the firm has not enough cash or marketable securities or risk-free debt to cover the cost of the project.
is greater than today’s i.e., the \textit{go} event is characterized by

\[
\tilde{x} < \frac{V_{go}}{V_{go} + k}(\tilde{x} + (1 + r)k) \quad \Leftrightarrow \quad \tilde{x} < (1 + r)V_{go}
\]  

(5.3)

i.e., current assets are not very valuable. As intuition told us, it is not a good idea to sell valuable assets (large realized \( x \)) and indeed, the firm will rationally pass the opportunity to launch the new project. Since the \textit{no} event is characterized by \( \tilde{x} > (1 + r)V_{go} \), we can check\(^{12}\) that

\[
V_{no} = \mathbb{E}[\tilde{x}|\tilde{x} > (1 + r)V_{go}] > (1 + r)V_{go} > V_{go}
\]  

(5.4)

i.e., the decision to go is “bad news” and is chastised by the market. This explains why stock price falls when a firm announces a new emission of equity.

Beyond the under-pricing issue which is a consequence of the revelation of private information by the firm to the capital market, we have an efficiency problem: a positive NPV project will not always be implemented because the \textit{no} event has a positive probability. If the firm could sell its old assets it would credibly reveal to the market the value of \( \tilde{x} \); then it would not suffer under-pricing and efficiency would be restored since the new investment would always occur. Indeed, the fair value of the actual shares would be \( V_{go} = \mathbb{E}[\tilde{x}|\text{go} \cup \tilde{x}] + r k = \tilde{x} + r k \) since the market information contains the decision to go and the true value of the firm’s current assets. Now, the condition \( \tilde{x} < (1 + r)V_{go} \) is always true for \( r > 0 \) i.e., every new project of positive NPV is undertaken. The value of the firm after the “go” decision would increase from \( \tilde{x} \) to \( \tilde{x} + r k \) because this time, the market rewards the decision to go on with the new project.

One could nevertheless wonder why the decision to issue equity could not be the “good” signal that the firm has encountered a new project of positive NPV? The reason has to do with the possibility that the rate of return \( r \) is itself uncertain. The firm will never undertake an investment with \( r < 0 \) because the capital would be better invested in riskless bonds. Yet for \( r = 0 \), the firm will go

\(^{12}\) Regarding existence of \( V_{no} \) and \( V_{go} \) it is enough to assume an infinite tail for the statistical distribution of \( x \). This implies that \( \mathbb{E}[\tilde{x}|\text{go}] = \mathbb{E}[\tilde{x}|\tilde{x} < (1 + r)V_{go}] \) is an increasing concave function of \( V_{go} \), thus the equation \( v - r k = \mathbb{E}[\tilde{x}|\tilde{x} < (1 + r) v] \) has a unique solution in \( v \) so that \( V_{go} \) is uniquely determined; it is increasing in the project size \( k \). Now, \( V_{no} = \mathbb{E}[\tilde{x}|\tilde{x} > (1 + r)V_{go}] \) is also uniquely determined.
on whenever \( \tilde{x} < V_{go} \), so that the equity issue could still be bad news.

The conclusion generally drawn from the previous model is called the “pecking order” theory stating that a firm should finance its projects internally before recurring to debt; equity should be used as a last resort only.

### 5.1.4 Signaling Quality

We leave aside the value of current assets and inquire the pricing of new assets (new projects). When the market is unable to distinguish the good projects from bad ones, the equity of good firms end up being underpriced which in turn might lead these firms to under-invest. Leland and Pyle (1977), in a model replicating Spence (1973) to the current setting, show that the entrepreneur can alleviate this information problem by retaining a large fraction of the firm’s equity to convince the market of its intrinsic quality. This behavior leans on the well known fact that the founder is human, thus she is risk averse and prefers to replace the risky cash flow of her firm by the certain return of an equity sell. Now, if the entrepreneur keeps most of her equity, it must be true that the return is high enough to compensate her for the risk she is bearing. Lenders therefore value more the few shares put for sale.

To demonstrate this claim, we consider a population of entrepreneurs displaying constant absolute risk aversion and facing normally distributed risk. As shown in §1.1.3, each maximizes \( \mathbb{E}[\tilde{w}] - \frac{1}{2} \rho \mathbb{V}[\tilde{w}] \) where \( \mathbb{E} \) denotes expectation and \( \mathbb{V} \) the variance of the random income \( \tilde{w} \). Each entrepreneur owns a technology of two possible types, good (\( g \)) or bad (\( b \)), characterized by a random future cash flow \( \tilde{x}_i \) for \( i = g, b \); the distributions have distinct expectations \( \mu_g > \mu_b = 0 \) (w.l.o.g.) but the same variance \( \sigma^2 \); we can thus w.l.o.g. scale this parameter in order to eliminate the constant multiplier \( \frac{1}{2} \rho \) from further formulas. After selling \( (1 - \alpha) \% \) of its equity for an amount \( v_i \) an entrepreneur of type \( i = g, b \) will have income \( \tilde{w}_i = \alpha \tilde{x}_i + v_i \) with \( \mathbb{E}[\tilde{w}_i] = \alpha \mu_i + v_i \) and \( \mathbb{V}[\tilde{w}_i] = \alpha^2 \sigma^2 \), thus her expected utility is \( u_i(\alpha) = \alpha \mu_i + v_i - \alpha^2 \sigma^2 \).

If there were no asymmetry of information, the market would always distinguish the two technologies and being risk neutral, it would pay \( v_i = \mu_i \). It is then obvious that each risk averse entrepreneur would be better off selling all her equity to the market in order to maximize \( u_i \); this would result in the optimal risk-sharing allocation. In the more realistic situation where the market
cannot distinguish the two technologies, there are two possible equilibria called 
“pooling” and “separating”. If the two types of entrepreneurs behave in a similar 
fashion, they leave the investors in the fog, the latter therefore put a single price 
for any equity and there is pooling of the types. If the two types of entrepreneurs 
behave in a very distinct manner, they might succeed to convince investors that 
one behavior is typical of a good firm and the other of the bad firm; there is separ-
ation of the types.

In the first case, the single price $v$ is an average between the value of a good 
and bad firm, thus a good firm is underpriced (and a bad one overpriced). In 
the second case, the market is able to price each type of equity at its real value; 
if we denote $\alpha_i$ the share kept by an $i$-entrepreneur, we have $v_i = \mu_i$ for $i = g, b$. Compared to pooling, separation is desired by a good firm but feared by a 
bad one so that the former will try to force it and the latter will try to avoid it. 
Wise people say that “talk is cheap” which means that the only credible behavior 
available to an entrepreneur to signal the quality of her firm passes through the 
decision to sell more or less of her equity. The incentive condition to avoid that 
a bad firm pretends to be a good one is that her utility when lying (i.e., keeping 
$\alpha_g$ as if she were a good firm) is lesser than when telling the truth (i.e., keeping 
$\alpha_b$):

$$u_b(\alpha_g) \leq u_b(\alpha_b) \iff (1 - \alpha_g)\mu \leq \sigma^2(\alpha_g^2 - \alpha_b^2)$$

(5.5)

If the bad firm cannot micmick the good one then it is better off selling all of her 
equity to eliminate risk, hence we need only consider (5.5) with $\alpha_b = 0$ so that 
the condition becomes after some algebraic manipulations

$$\alpha_g \geq \bar{\alpha} \equiv -\mu + \sqrt{\mu(\mu + 4\sigma^2)} \quad (5.6)$$

Conversely a good firm should not be tempted to mimic a bad one by selling all 
of its equity i.e.,

$$u_g(0) \leq u_g(\alpha_g) \iff \alpha_g \leq \tilde{\alpha} \equiv \sqrt{\frac{\mu}{\sigma^2}}$$

(5.7)

Algebraic manipulations\(^{13}\) enable to show that $\underline{\alpha} < \tilde{\alpha}$, hence both conditions are 
compatible for $\alpha \in [\underline{\alpha}; \tilde{\alpha}]$. Now given that risk-sharing is the motive for selling

\(^{13}\) $\underline{\alpha} \leq \tilde{\alpha} \iff \sqrt{\mu(\mu + 4\sigma^2)} - \mu \leq 2\sigma^2 \sqrt{\mu/\sigma^2} = 2\sqrt{\mu\sigma^2}$
equity in the first place, a good firm will sell the maximum and retain no more than $\alpha\%$ of its equity to signal credibly its quality to market investors (this also proves that only bad firms are tempted to mimic good ones).

Given our convention, the parameter $\mu$ measures the extent of asymmetry of information and it is not difficult to check that $\alpha$ is increasing with $\mu$; hence, the stronger the information asymmetry, the greater the risk the entrepreneur has to bear (by retaining more shares). When applied to the real world, our findings predict that in fast growing industries (e.g., IT services) which display large asymmetries of information, manager/founders of firms retain more equity than managers of firms in mature sectors (e.g., traditional industry) where there is little asymmetry of information.

5.2 Agency Cost of Debt Finance

We now switch instrument and concentrate on debt. If capital is raised from debt with face value $d$ to be repaid at the interest rate $r$, the entrepreneur's profit is $R(d) - (1 + r)d$ and the FOC of maximization is $R_m = 1 + r$. Assuming a perfect financial market (no transaction cost) the price of money is the same for borrowing and lending i.e., $r = 0$ given our previous convention regarding the PV of $R$. In that case, the incentives to invest are adequate since the optimal debt choice is $d = k^*$, the efficient investment.

In the absence of uncertainty (debt is riskless), lump sum finance such as debt is efficient in the sense that the incentives of the entrepreneur to invest are not distorted (with respect to the case where she does not need external capital). This section will precisely introduce realistic features such as uncertainty or premium for borrowers to test the robustness of this efficiency result. Asset substitution generates excessive investments on the part of the entrepreneur while debt overhang works in the opposite direction; we also present a model blending the two effects to understand better under which conditions one is more likely to take place. Next, we look at credit rationing, an adverse selection phenomenon induced by debt finance.

\[
\begin{align*}
\Leftrightarrow & \quad 4\mu\sigma^2 \geq \left(\sqrt{\mu(\mu + 4\sigma^2)} - \mu\right)^2 = \mu(\mu + 4\sigma^2) + \mu^2 - 2\mu\sqrt{\mu(\mu + 4\sigma^2)} \\
\Leftrightarrow & \quad 2\mu\sqrt{\mu(\mu + 4\sigma^2)} \geq \mu(\mu + 4\sigma^2) + \mu^2 - 4\mu\sigma^2 = 2\mu^2 \\
\Leftrightarrow & \quad \sqrt{\mu(\mu + 4\sigma^2)} \geq \mu \Leftrightarrow \mu(\mu + 4\sigma^2) \geq \mu^2 \Leftrightarrow \mu + 4\sigma^2 \geq \mu \Leftrightarrow 4\sigma^2 \geq 0!
\end{align*}
\]
5.2.1 Asset Substitution and Free Cash Flow

Intuition

Jensen and Meckling (1976) is again the seminal article that first called the attention on the asset substitution induced by debt (aka. the over-investment effect of debt). In the presence of limited liability and future uncertainty, the entrepreneur is indifferent with respect to the size of bankruptcy and only cares for the happy times where she can pay her debt and keep all remaining profits. This leads her to undertake too risky and even unprofitable investments.

To get the idea suppose that as a firm’s manager you borrow 60 to finance a new project. You can either invest in a very secure idea yielding 100 and make a sure profit of 40, or, take a gamble with a risky project that either yield 180 or 0. Under the risky alternative you either win 120 or 0 since you are protected by limited liability, hence on average you make 60 which turns you into a risk lover. Notice that you selected the inefficient project since its average return is only 90 < 100. This phenomenon is not limited to finance. In sports like football where matches have fixed duration, it is frequent to see the lagging team take increasingly more risk in a desperate attempt to catch up, the result being either an unlikely victory or a more presumable overwhelming defeat. The rationale is quite obvious whenever the ultimate goal is to achieve victory: given the current score and the current field strategy, defeat is almost certain, thus it cannot worsen to change the strategy into a more aggressive one that will increase the probability of winning although it also increase the probability of losing big (but who cares since the team would have been eliminated anyway).\(^{14}\)

Model

To see the asset substitution formally, we modify the previously used DRS technology so as to account for market price uncertainty; the profit is $\pi = \hat{p}R(k) - k$ where the market price $\hat{p}$ is random with a distribution function $H$ and mean

\(^{14}\)Football in major tournaments is rather like the war of attrition where each team waits for an error or a risk taking by the other side to counter-attack and secure an advantage.
\[ \mathbb{E}[\hat{\rho}] = 1 \] (so as to maintain our initial convention). The expected NPV is
\[ \pi^*(k) = \int_0^{+\infty} (p R(k) - k) \, dH(p) = \mathbb{E}[\hat{\rho}] R(k) - k = R(k) - k \]
so that the optimal investment is again the efficient level \( k^* \) solving \( R_m(k) = 1 \).

We consider a risk-neutral entrepreneur who has neither personal wealth nor is able to emit new equity to finance her project; she is thus forced to finance it entirely with debt. Following our convention, the risk-free interest rate is \( r_0 = 0 \). We shall show that the uncertainty over the market price generates an agency cost in the sense that the entrepreneur’s optimal decision is now to over-invest (with respect to the first best decision \( k^* \) that a wealthy entrepreneur would pick).

As we explained in the previous examples, whenever the realized cash-flow \( \hat{\rho} R(k) \) does not cover the debt obligation \( k \), the entrepreneur defaults and walks out with a zero profit but no penalty since she is protected by limited liability. The cut-off level below which defaults takes place is \( \hat{\rho} \equiv \frac{k}{R(k)} \). At the time where she must decide on her investment, she realizes that only the good times matter i.e., her expected profit is
\[ \hat{\pi} = \int_{\hat{\rho}}^{+\infty} (p R(k) - k) \, dH(p) = (R(k) \mathbb{E}[\hat{\rho} | \hat{\rho} \geq \hat{\rho}] - k) \left( 1 - H(\hat{\rho}) \right) \] (5.8)

We see that, with respect to a self-financed entrepreneur, she earns more because although \( \hat{\pi} \) and \( \pi^* \) are the integral of the same surplus, the former goes over positive surpluses only. To ascertain the incentives towards investment, we look at the FOC of profit maximization:
\[ 0 = \frac{\partial \hat{\pi}}{\partial k} = \int_{\hat{\rho}}^{+\infty} (p R_m(k) - 1) \, dH(p) + \underbrace{(\hat{\rho} R(k) - k)}_{=0} h(\hat{\rho}) = (R_m(k)\mathbb{E}[p | p \geq \hat{\rho}] - 1) \left( 1 - H(\hat{\rho}) \right) \] (5.9)
\[ \Leftrightarrow R_m(k) = \frac{1}{\mathbb{E}[\hat{\rho} | \hat{\rho} \leq \hat{\rho}]} \geq \frac{1}{\mathbb{E}[\hat{\rho}]} = 1 \] (5.10)
i.e., the debt-financed entrepreneur over-invests (recall the comparative static
This behavior generates default with a positive probability so that lenders do not recoup their loan on expectation; they will have to ask for a risk premium \((r > 0)\) whose effect is to induce under-investment (cf. §5.1.1 and next paragraph). Nevertheless, this (second) indirect effect remains dominated by the (first) direct one as long as the interest rate is determined by the participation constraint of lenders (they do not make a loss on expectation).

To prove that claim, we use the fact that
\[
R_m(k^*) = 1 = \mathbb{E}[\hat{p}] 
\]

The repayment to the lender being \(\hat{b} = \min\{\hat{p}R(k), (1 + r)k\}\), we can write
\[
\frac{\mathbb{E}[b]}{k} = \frac{R(k)}{k} \int_{0}^{\hat{p}} p \, dH(p) + (1 + r) \left( 1 - H(\hat{p}) \right) 
\]

If the participation constraint \(\mathbb{E}[b] \geq k\) is binding, then observe from (5.12) that
\[
1 < \frac{R(k)}{k} \int_{0}^{\hat{p}} p \, dH(p) + (1 + r) \left( 1 - H(\hat{p}) \right) = H(\hat{p}) + (1 + r) \left( 1 - H(\hat{p}) \right)
\]
hence \(r > 0\).

Next, we use the fact that \(\frac{R(k)}{k} > R_m(k)\) to deduce that the first term of (5.12) is greater than the first term (5.11). Given that (5.11) and (5.12) are equal, it must be the case that seconds terms are inversely ordered i.e.,
\[
(1 + r) \left( 1 - H(\hat{p}) \right) < R_m(k^*) \int_{\hat{p}}^{+\infty} p \, dH(p) \iff (1 + r) < R_m(k^*)\mathbb{E}[p | p \geq \hat{p}] 
\]

The debt-financed entrepreneur earns marginally more. Indeed, the average return \(\frac{R(k)}{k}\) of a DRS technology is greater than its marginal \(R_m(k)\), thus \(\frac{1}{\hat{p}} = \frac{R(k)}{k} > R_m\) implies that \(1 > \hat{p}R_m(k)\) thus \(pR_m(k) - 1 < 0\) for \(p \in [0; \hat{p}]\) and therefore \(\frac{\partial\pi}{\partial k} > \int_{0}^{+\infty} \left( pR_m(k) - 1 \right) dH(p) = \frac{\partial\pi^*}{\partial k}\).
Taking into account the risk premium of investors, \( \hat{\pi} = \int_{\hat{p}}^{\infty} (pR(k) - (1 + r)k) \, dH(p) \)

\[
\frac{\partial \hat{\pi}}{\partial k} \bigg|_{k^*} \propto R_m(k^*) \mathbb{E}[p|p \geq \hat{p}] - (1 + r) > 0
\]

meaning that the optimal investment \( \hat{k} \) is greater than the efficient one \( k^* \).

Over-investment in this simple model has a multiplier effect on the rate of return of the project, it thus acts as a gamble (risk-taking) with respect to the efficient level.

**Free Cash Flow**

Jensen (1986) identifies another source of moral hazard, arguing the financial resources at the disposal of managers, the so-called free cash flow leads them towards over-investment. Indeed, if managers have the power to decide on the use of cash flow, they are likely to waste it in projects with low return (negative NPV), into acquisitions to derive more power or worse, into perks (e.g., build a pool at home with the firm’s money). Hence everything should be done to limit the quasi-rents at the disposal of managers in order to reduce their uncontrolled waste and force them to incur the monitoring of the capital markets when they must raise new capital to finance investments. This problem is more important in sectors that generate large cash flows but have low growth prospects like regulated monopolies. According to Jensen (1986), it would be therefore desirable to force managers to pay large dividends to shareholders whenever they have free cash flow; but since promises are not credible (recall that “talk is cheap”) one way of achieving this objective is to emit debt in exchange of a repurchasing of shares. This way, a former shareholder becomes a debtholder and has the right to take the firm into bankruptcy court if it defaults on debt service. This swap of securities makes debt a credible substitute for dividends and reduces the agency costs of free cash flow (inefficient over-investment). Empirical evidence support this view since share prices tend to respond positively to debt-equity swaps indicating that investors interpret these decisions as value enhancing.
5.2.2 Debt Overhang

Option to Invest

A radically different point of view with respect to the role of debt in agency situations is taken by Myers (1977) who claims that an entrepreneur faces a debt overhang problem because at any point in time, investments are optional choices to be undertaken. So, whenever the net return of a project is lesser than the outstanding debt service, it is better for the entrepreneur to drop it and go bankrupt since she is protected by limited liability; somehow, she free-rides on the investor. The overall effect is under-investment since some valuable projects are not implemented.

To explain formally this phenomenon, we add a fixed cost $\tilde{c}$ whose level is not yet known by the entrepreneur at the time where she decides on the investment; its distribution function is $H$.\textsuperscript{16} If the entrepreneur gets financed exclusively by debt at the interest rate $r$ (recall that the risk-free rate is zero under our convention) her random profit is $\tilde{\pi} = R(k) - \tilde{c} - (1 + r)k$ so that she is forced into bankruptcy whenever $\tilde{c} > \hat{c} \equiv R(k) - (1 + r)k$ i.e., whenever the fixed cost realization is large. The cut-off $\hat{c}$ must be positive for otherwise the technology would be useless (at the current interest rate), but this also means that default occurs with positive probability. Hence, the expected repayment to lenders is strictly lesser than $(1 + r)k$, so that $r$ must be positive to compensate their initial investment of $k$. The entrepreneur’s expected profit being

$$\mathbb{E}[\tilde{\pi}] = \int_{0}^{\hat{c}} (R(k) - (1 + r)k - c) \, dH(c) \quad (5.14)$$

we have

$$\frac{\partial \mathbb{E}[\tilde{\pi}]}{\partial k} = \frac{R_m(k) - (1 + r)}{H(\hat{c})} \quad (5.15)$$

thus the optimal investment solves $R_m = 1 + r$ and since $R_m$ is decreasing and $r > 0$, it involves under-investment.

To understand the difference with the previous over-investment result, notice that unlike a multiplicative shock like price, an additive shock like fixed

\textsuperscript{16}Alternatively, the random component could be the cash-flow generated by current assets out of which the entrepreneur will finance part of her new investment.
cost has no effect on the marginal productivity, thus does not distort incentives. However, both generate default which is risk from the point of view of lenders; this means that they demand a positive risk premium whose (indirect) effect is to dampen incentives to invest.

Empirically, firms in mature sectors (e.g., heavy industries) have free cash flow and little investment opportunities so that they tend to invest in negative NPV projects. The reverse holds for firms in high growth sectors (e.g., IT services) who lack funds but not ideas and cannot implement as many as efficiency would command. To conclude, over-investment should be more severe in mature industries while under-investment should be more severe in high growth industries.

A rejoinder

Loosely speaking our previous models conclude for over-investment occurs if the risk is multiplicative (e.g., market price uncertainty) and under-investment if the risk is additive (e.g., fixed cost uncertainty). The following adaptation\footnote{This presentation is inspired by Berkovitch and Kim (1990) and Yossi Spiegel's teaching notes.} draws a clearer frontier among the two effects by blending together the original arguments of Myers (1977) and Jensen and Meckling (1976): over-investment is likely to happen if the firm's realized cash-flow is greater than expected (free cash flow) while under-investment takes place in the reverse situation (debt overhang).

Assume that the entrepreneur uses long-term debt to finance a project developing in two stages, with a promise to repay $d$ at the end. Ex-ante, she raises funds to run her normal business and to perform R&D with a view to develop a better technology. At the interim period, the current assets yield a cash-flow $x$, a part of which $k$ can be used to invest into the new technology $R$ which is now available; the efficient investment $k^*$ solving $R_m(k) = 1$ can be undertaken at the interim period.

What creates a moral hazard problem at the interim stage (when $k$ is chosen) is the market price uncertainty that plagues every period: from the ex-ante point of view, the interim cash flow $x$ is random, hence there is no way to tune the ex-ante investment so as to obtain exactly $x = k^*$ which is necessary in order to re-invest optimally into the new technology. Now, at the interim period, the
entrepreneur disposes of a realized cash flow $x$ and must decide how much to invest in her new technology knowing that the ex-post market price $\tilde{p}$ will also be random (with unit mean).

If the realized interim cash flow is $x < k^*$, then the entrepreneur is forced to put up money to invest at the desired level.\textsuperscript{18} Since she faces the debt overhang problem, she won’t put the missing $k^* - x$ but less, so there is under-investment. If, on the contrary, the interim cash flow is a generous $x > k^*$, then the entrepreneur has \textit{free cash-flow} and takes advantage of the existence of debt to gamble over the uncertainty of the final return; she undertakes excessive investment.

To derive precisely these results let us study the optimal investment of the entrepreneur in two polar cases:

\textbf{high cash-flow} by investing $k \leq x$, her final wealth is $v_h = \max\{0, \tilde{p}R(k) - d + x - k\}$.

\textbf{low cash-flow} by investing $k > x$ (putting up $k - x$ out of her pocket), her final wealth is $v_l = \max\{0, \tilde{p}R(k) - d\} + x - k$.

Let $p_h$ and $p_l$ be the solutions of $\tilde{p}R(k) + x - k = d$ and $\tilde{p}R(k) = d$. The respective interim expected wealth are thus

$$
\mathbb{E}[v_h] = \int_{p_h}^{\infty} (pR(k) - d + x - k) \, dH(p) \quad \text{if } k \leq x
$$
$$
\mathbb{E}[v_l] = x - k + \int_{p_l}^{\infty} (pR(k) - d) \, dH(p) \quad \text{if } k > x
$$

The optimal investments in each case are $k_h$ and $k_l$ solving

$$
R_m(k_h) = \frac{1 - H(p_h)}{\int_{p_h}^{\infty} p \, dH(p)} = \frac{1}{\mathbb{E}[\tilde{p} | \tilde{p} \geq p_h]} \quad \text{if } k \leq x
$$
$$
R_m(k_l) = \frac{1}{\int_{p_l}^{\infty} p \, dH(p)} \quad \text{if } k > x
$$

Basic probability tell us that

$$
\int_{p_l}^{\infty} p \, dH(p) < \int_{0}^{\infty} p \, dH(p) = 1 = \mathbb{E}[\tilde{p}] < \mathbb{E}[\tilde{p} | \tilde{p} \geq p_h]$$

so that since $R_m$ is decreasing, we have the ranking $k_l < k^* < k_h$.

\textsuperscript{18}She can emit new equity paying a fixed dividend to avoid moral hazard.
A bit of logic is required to conclude. If the cash flow realization is large \((x > k_h)\) then the optimum investment is \(k_h > k^*\), there is thus over-investment (yet never by more than \(k_h - k^*\)). Conversely, if the cash flow is low \((x < k_l)\) then the optimum investment is \(k_l < k^*\), there is under-investment (yet never by more than \(k^* - k_l\)). Finally, when the cash flow is intermediate, it is entirely invested.\(^{19}\)

There is thus either a moderate under or over investment depending on where \(x\) falls with respect to \(k^*\).

5.2.3 Debt as the Optimal Security

Intuition

We tackle here the problem of optimal security design i.e., finding the most efficient contract an entrepreneur and an investor might sign. Recall for instance that equity leads to under-investment by the entrepreneur either in physical or human capital because she is forced to share part of the future profits with the investor. Under the reasonable assumption of limited liability for the entrepreneur, Innes (1990) shows that debt is optimal to promote effort in situations of potential moral hazard.

The result builds on two simple observations. When the cash-flow is low, the investor gets to keep all of it so that the entrepreneur’s motivation towards effort is minimal. On the contrary, when the cash-flow is large enough to cover the debt obligation, the entrepreneur is the residual claimant of any increase in cash flow, thus is optimally motivated towards effort. Obviously, efficiency corresponds to zero debt so as to make sure the entrepreneur is always the residual claimant. The need for external finance will necessarily introduce a distortion in the sense that for some values of the cash-flow realization, the entrepreneur will receive only a fraction of this cash-flow; as a consequence, she will have incentives to under-invest, thereby generating an inefficient outcome, deemed a “second best”.

Our original question boils down to decide where to put these distortions. It turns out that debt is optimal because it concentrates the distortion on low levels of cash-flow which represent small prizes, thus small disincentives while it makes the entrepreneur residual claimant for large cash-flow which represent

\(^{19}\)Conditional on the cash flow realization, the optimal investment is a corner solution at \(x\).
large prizes giving large incentives toward effort. On Figure 5.3, we show four reimbursement rules: $\gamma_d$ is the debt rule corresponding to loan $d$ (bold curve), $\gamma_\alpha$ is the equity rule corresponding to the sale of a share $\alpha$ of future profits, $\gamma$ is the strange rule where there is no repayment until cash-flow reaches a minimum, then the repayment increases faster than cash-flow until it achieves a maximum; lastly $\hat{\gamma}$ is a weird repayment rule that does not make any economic sense but which is nevertheless imaginable. Clearly, the debt rule is the closest to the diagonal for low cash-flows and then the farthest away for large cash-flows; this means that when compared to another rule, the debt rule is firstly above then permanently below.

![Figure 5.3: Repayment Rules](image)

**Model**

To prove formally this result, consider a penniless entrepreneur investing $k$ into a project. The future cash-flow is a random variable $\tilde{x}$ such that $H(k, x) = Pr(\tilde{x} \leq x \mid k)$, the probability to observe a result lesser than $x$ depends on the investment $k$. For any function $f$, we denote $\mathbb{E}[f(\tilde{x}) \mid k] = \int f(x) dH(k, x)$.

An investor pledges $k$ in exchange for a repayment rule $\gamma$ function of the future cash-flow. Popular rules shown on Figure 5.3 are debt with $\gamma_d(x) = \min\{x, d\}$ or equity with $\gamma_\alpha(x) = \alpha x$. The restrictions for an admissible rule are $\gamma(x) \leq x$ because the entrepreneur is protected by limited liability (she has no collateral to
pledge to her creditor) and $\gamma' \geq 0$. To understand this later property, imagine that $\gamma$ is decreasing at some level of cash-flow $x$; the creditor could then artificially reduce the available cash flow down to $x'$ by calling a costly external audit and get a contractual repayment $\gamma(x') > \gamma(x)$. The actual repayment rule would, at most, be flat but never decreasing. The investor’s expected repayment is $\pi(\gamma, k) = \mathbb{E} [\gamma(\tilde{x}) | k]$ so that the entrepreneur’s expected profit is $\pi(\gamma, k) = \mathbb{E} [\tilde{x} - \gamma(\tilde{x}) | k] - k$.

Assume that the agreed rule $\gamma$ is optimal and is not a debt rule. The entrepreneur elicits the investment $k$ maximizing $\pi(\gamma, k)$. i.e., solving $R_m(k) = 1 + \frac{\partial \mathbb{E} [\gamma(\tilde{x}) | k]}{\partial k}$. Since $\mathbb{E} [\gamma_d | k]$ is increasing with leverage $d$, there exists a unique debt level $d$ such that $\mathbb{E} [\gamma | k] = \mathbb{E} [\gamma_d | k]$ By construction of a debt rule, the difference $f(x) \equiv \gamma(x) - \gamma_d(x)$ is negative for small $x$, zero for some $y$ value and then becomes positive for $x > y$ (check on the right panel on Figure 5.3); it displays the single crossing property (SCP).

We now assume that a higher effort induces a change in cash-flow distribution satisfying (MLRP)20 so that we can conclude (after Milgrom (1981)) that the expectation of the random variable $f(\tilde{x})$ is increasing with the investment $k$. In other words, its derivative with respect to $k$ is positive which given the definition of $f$ reads

$$\frac{\partial \mathbb{E} [\gamma | k]}{\partial k} > \frac{\partial \mathbb{E} [\gamma_d | k]}{\partial k} \quad (5.16)$$

Property (5.16) of the cash-flow distribution means that a small additional investment increases the expected debt repayment less than under any other rule. Thus, this small additional investment increases the entrepreneur’s expected profit faster under debt finance than under any other rule i.e.,

$$\frac{\partial \pi(\gamma_d, k)}{\partial k} > \frac{\partial \pi(\gamma, k)}{\partial k} \quad (5.17)$$

By optimality of $k$ under the rule $\gamma$, the RHS of (5.17) is nil, meaning that under the new debt rule $\gamma_d$, the entrepreneur can increase her expected profit by choosing an investment $\hat{k} > k$ which is welcomed by the investor since $\mathbb{E} [\gamma_d | \hat{k}] > \mathbb{E} [\gamma_d | k] = \mathbb{E} [\gamma | k]$. This fact ends the proof that the original rule $\gamma$ was not opti-

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20 Assuming that the law $H(k, x)$ admits a density $h(k, x)$, the change from $k$ to $\hat{k}$ satisfies the monotone likelihood ratio property (MLRP) if $\hat{k} > k \Rightarrow \frac{h(\hat{k}, x)}{h(k, x)} > \frac{h(k, x)}{h(\hat{k}, x)}$ in $x$. The family of distributions $h(k,)$ is said to satisfy MLRP if the previous property is true for all parameter values.
Risk Aversion

If we wish to take into account that the entrepreneur is most often risk-averse (relative to the investor), Matthews (2001) shows that debt optimality continues to hold if the original contract can be renegotiated by the entrepreneur after she invested but before the cash flow is realized. Consider a non debt candidate optimum $\gamma$ that is renegotiated towards a contract $\phi$. The objective of the entrepreneur at that point is to eliminate risk which would require $\phi(x) = x - cte$ but this would violate the investor limited liability, thus the entrepreneur would choose something akin to the inverse of debt with $\phi(x) = \max\{0, x - cte\}$ tuning the constant so as to generate acceptance by the investor. The problem then is that this new final contract generates poor incentives to invest (it’s the opposite of debt!) and therefore cannot raise a lot of money from the investor. A midpoint will have to be struck to preserve investment incentives with respect to risk sharing.

Since the entrepreneur final payoff is $\pi(\phi, k) = \mathbb{E}\left[u(\tilde{x} - \phi(\tilde{x}) - k)|k\right]$ for some concave function $u$, the investor expects the investment $k^*$ to maximize $\pi(\phi, k)$; he will thus accepts $\phi$ only if $\mathbb{E}\left[\phi|k^*\right] \geq \mathbb{E}\left[\gamma|k^*\right]$. If now the initial contract is changed for a debt one $\gamma_d$ such that $\mathbb{E}\left[\gamma_d|k^*\right] = \mathbb{E}\left[\gamma|k^*\right]$ then the entrepreneur can still invest $k^*$ and offer $\phi$ in renegotiation which proves she can’t lose from the change towards debt. We still need to show that the investor won’t fear a change of investment that is bad for him, thus making the initial offer uninteresting.

Let $\phi_d$ and $k_d$ be optimal after $\gamma_d$ i.e., $k_d$ maximize $\pi(\phi_d, k)$ and $\mathbb{E}\left[\phi_d|k_d\right] \geq \mathbb{E}\left[\gamma_d|k_d\right]$. If, on the one hand, $\pi(\phi_d, k_d) = \pi(\phi, k^*)$ then $(\phi, k^*)$ is optimal after $\gamma_d$ i.e., the change to debt does not change the final contract nor investment, thus the investor does not lose. If, on the other hand, the change to debt is profitable for the entrepreneur then she invests more\textsuperscript{21} which is unilaterally good for

\textsuperscript{21}Our claim is the contraposition of the following property: for any $k < k^*$, $\pi(\phi, k) \geq \pi(\phi_d, k)$. To prove the property, we use MLRP: $\mathbb{E}\left[\gamma_d|k^*\right] = \mathbb{E}\left[\gamma|k^*\right] \Rightarrow \mathbb{E}\left[\gamma_d|k\right] \geq \mathbb{E}\left[\gamma|k\right]$. This says that every renegotiation offer $\phi$ acceptable after $\gamma_d$ is also acceptable after $\gamma$ (both conditional on investment $k$), hence the maximum of $\pi$ for renegotiation following $\gamma$ cannot be less than the maximum of $\pi$ for renegotiation following $\gamma_d$. 

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5.2.4 Credit rationing

Credit rationing as well as unemployment are important preoccupations for macroeconomists but also for micro-economists since these long lasting disequilibrium phenomena fail to be explain by the classical general equilibrium theory. Inspired by Akerlof (1970)’s lemons model of the used car market (cf. §3.1.1), Stiglitz and Weiss (1981) show how asymmetries of information among borrowers and lenders can create a market imperfection that limits the volume of credit and endogenously generate credit rationing.

In a credit market, like any other market, demand is decreasing in the nominal interest rate while supply is increasing with the effective interest rate (return on unit loan). In a perfectly competitive market where asymmetries of information are absent, the two kinds of rate are identical. The novelty is to demonstrate how uncertainty about the quality of lenders can make a difference between the nominal and realized interest rates. More precisely, it might be the case that the realized interest rate reaches a maximum such that the corresponding (larger) nominal rate generates excess demand for credit. There is credit rationing because no lender agrees to loan more as he knows this would only lower his effective return.

The story behind this phenomenon builds on the well known positive correlation between leverage and default. Understanding this relationship, lenders tend to believe that firms asking much credit are signaling a high probability of default. This in turn leads them to ask for a large risk premium. The problem with this attitude is that it may affect the average quality of applicants (adverse selection) and their behavior once financed (moral hazard). As we shall demonstrate afterwards, raising the nominal rate does not reduce the demand for credit in an even manner because the safest entrepreneurs drop out so that the pool of remaining applicants are of lower intrinsic quality and worse still, they take more risk than ever. These negative effects diminish the effective interest rate, so much that it might be the case that they outweigh the original nominal increase.

The adverse selection effect is quite similar to the asset substitution effect: riskier projects are more profitable on expectation whenever the entrepreneur...
uses debt and is protected by limited liability. To see this formally, consider two projects looking for the same funding $k$ with random cash-flows $\tilde{x}$ and $\tilde{y}$, that have the same expected value i.e., $\mathbb{E}[\tilde{y}] = \mathbb{E}[\tilde{x}]$. Since the repayment of debt $R(\tilde{x}) = \min\{(1 + r)k, \tilde{x}\}$ is linear then constant, it is concave, thus the entrepreneur’s profit, $\pi(\tilde{x}) = \tilde{x} - R(\tilde{x})$, is convex. Rothschild and Stiglitz (1970) introduce a statistical notion of riskiness such that if $\tilde{y}$ is more risky than $\tilde{x}$, then $\mathbb{E}[\pi(\tilde{y})] \geq \mathbb{E}[\pi(\tilde{x})].^{22}$ When a lender increases the nominal rate $r$, firm profits are lowered, hence also fall on expectation so that the less risky project $\tilde{x}$ drops out i.e., cease to demand credit. This means that the average riskiness of applicants increases (only $\tilde{y}$ remains) and therefore the average expected repayment drops (at constant interest rate).^{23}

If there are only two classes of risk, safe and risky people, there is a cut-off nominal rate, say $\bar{r}$ where safe people drop out; at that point the average repayment falls much more than the gain generated by the nominal rate increase. In other words, the realized interest rate reaches a maximum at $\bar{r}$. The optimum for lenders is therefore to offer the highest nominal rate that guarantee participation from all entrepreneurs i.e., $\bar{r}$. This also means that the supply of funds has reached a maximum because supply is an increasing function of the effective rate. Yet this optimal rate proposed by lenders generates a demand for funds larger than the supply so that credit rationing occurs.

Regarding moral hazard, notice that after a nominal rate increase, the structure of profits becomes more convex,^{24} thus motivate entrepreneurs to gamble and choose more risky projects thereby worsening the expected repayment to lenders and the realized return on their loans. Indeed, we have $\mathbb{E}[\pi(\tilde{x})] = \int_{(1+r)d}^{\infty}(x - (1 + r)d) dH_1(x)$ where $H_1$ is the distribution function of the random variable $\tilde{x}$. By the definition of the cut-off where the entrepreneur defaults,

$$\frac{\partial \mathbb{E}[\pi(\tilde{x})]}{\partial r} = -(1 - H_1((1 + r)d)) d < 0$$

hence

$$\frac{\partial \mathbb{E}[\pi(\tilde{x})]}{\partial r} < \frac{\partial \mathbb{E}[\pi(\tilde{y})]}{\partial r} \iff H_1((1 + r)d) < H_2((1 + r)d)$$

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22 The notion is second order stochastic dominance.

23 Since that $R(\tilde{x})$ is concave, $-R$ is convex, it is thus enough to apply the above characterization of riskiness to see that riskier applicants repay less on expectation.

24 Draw $\pi(\tilde{x}) = \max\{\tilde{x} - (1 + r)d, 0\}$ as a function of $\tilde{x}$ to check the effect of increasing $r$. 

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where $H_2$ is the distribution function of the random variable $\tilde{y}$. If, at the initial interest rate, the firm was indifferent between two projects $\tilde{x}$ and $\tilde{y}$ ($\pi(\tilde{x}) = \pi(\tilde{y})$) then after the increase of $r$, the project with a higher probability of default $\tilde{y}$ is preferred to the safer one (profit was reduced for both but less for $\tilde{y}$).

This Stiglitz and Weiss (1981) result is however highly sensitive to the nature of the uncertainty. If the screening process of lenders identifies returns but leaves doubts regarding the probability of success, de Meza and Webb (1987), show that too many projects are undertaken. The reason is quite simple: when the return in case of success $R$ is known the entrepreneur’s profit is increasing with $p$, thus the projects that ask for funding are those above the participation threshold, contrary to what happens in the Stiglitz and Weiss (1981) case. The reverse equilibrium sub-optimality then follows.

### 5.3 Managerial Incentives

Success in business is rarely immediate which means that an entrepreneur needs time to build experience and try different options before she can hope to hit the jackpot, so to say. We saw previously that equity finance had a demotivating effect but debt financing is not perfect either. Indeed, since cash-flow is likely to be low during the first years there is a serious possibility of defaulting on the debt obligations. Now, going bankrupt is a very dark prospect for the entrepreneur. Not only does she loses the prestige of her position but above all she loses all the human effort she invested in the firm; in other words, the human capital she amassed is complementary to the physical assets of the firm.

A similar albeit weaker argument holds for the manager of a dispersedly owned firm: failing to maximize profits in the presence of debt increases the probability of bankruptcy, thus the probability of losing one's job and the perquisites associated with it.$^{26}$

The first two works presented below use this observation to show that debt can alleviate agency problems of adverse selection and moral hazard. In the

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$^{25}$This critique is not given prominent space because the model does not generalize beyond the success-failure model used by these authors.

$^{26}$Other typical but also imperfect measures to align the objective of managers with that of shareholders are salary incentives (participation to profits) and stock-options.
third part we present the efficiency wage theory explaining how asymmetries of information force firm to pay higher than necessary wages (and also generate unemployment).

5.3.1 Debt as a Signal of Profitability

Ross (1977) in an early application of Spence (1973)’s signaling theory shows how the manager of a firm can use debt to signal the profitability of her firm.

In the absence of uncertainty and under complete information regarding profitability, the value of a firm is correctly assessed by the market and the financial structure (debt or equity) does not matter. If now the profitability or future cash-flow is a private information of managers, the market will price each firm at an average which means that a profitable firm will be underpriced. Most attempts to signal a high profitability to the market will be mimicked by lower quality firms. In that situation, debt can help a manager. To see this, imagine there are only two firms, one “good” and one “bad” with certain future cash flows $x$ and $y < x$. The “good” manager needs to emit debt $d \in [y; x]$ to avoid bankruptcy but make sure that an imitator would surely go bust. Then it must be the case that bankruptcy is a concern for her to tell the market that her choices reveal her desire to avoid bankruptcy. As we previously argued, this is naturally verified for an entrepreneur. As for a manager, she must tie her own remuneration in a significant way to the final cash flow of her firm to convince the market that she cares to avoid bankruptcy. Her remuneration contract might incorporate a share of the bankruptcy costs.

The formal model uses a continuum of types $\tau$ uniformly distributed over $[a; b]$. The future cash-flow of a type $\tau$ firm is a random variable $\tilde{x}_\tau$ uniformly distributed in $[0; 2\tau]$. Given a debt level $d$, the terminal value of a $\tau$-firm is $\tilde{x}_\tau$ if she successfully repays her debt (case $\tilde{x}_\tau \geq d$) and $\tilde{x}_\tau - L$ otherwise, $L$ being the cost of reorganization generated by defaulting on the debt obligation. The expected final value is thus computed as

$$V_1^T(d) = \int_0^d (\tilde{x}_\tau - L) \, dx + \int_d^{2\tau} \tilde{x}_\tau \, dx = \int_0^{2\tau} \tilde{x}_\tau \, dx - L \int_0^d \, dx = \tau - Ld / 2\tau$$

\footnote{This is the famous irrelevance theorem of Modigliani and Miller (1958).}
assuming \( d < 2\tau \) (debt lesser than maximum cash-flow).

Let us consider the remuneration \( w = \gamma_0 V_0 + \gamma_1 V_1 \). In equilibrium the choice \( d_\tau \) of a type-\( \tau \) firm must be optimal for the manager, hence \( \frac{\partial w}{\partial d} = 0 \Leftrightarrow \gamma_0 V_0' + \gamma_1 V_1' = 0 \Leftrightarrow V_0'(d_\tau) = \frac{\gamma_1 L}{2\gamma_0 \tau} \). Since in equilibrium types are revealed we have \( V_0(d_\tau) = \tau \) hence \( V_0'(d_\tau) d_\tau' = 1 \). Plugging into the previous equation we obtain \( d_\tau' = \frac{2\gamma_0 \tau}{\gamma_1 L} \), thus \( d_\tau = \frac{\gamma_0 \tau^2}{\gamma_1 L} + c \) where \( c \) is the integration constant. Taking into account the fact that the worst type will not emit any debt to eliminate the risk of bankruptcy, we derive \( d_\tau = \frac{\gamma_0 (\tau^2 - a^2)}{\gamma_1 L} \). Lastly we must check that our initial assumption \( d < 2\tau \) is satisfied which requires eliciting \( \gamma_1 \) large in front of \( \gamma_0 \) i.e., the manager's remuneration must strongly depend on the future where bankruptcy might happen to credibly transmit information to the market as regard the type of her firm.

5.3.2 Debt as a Signal of Obedience

A moral hazard issue tantalizing financiers is the possibility that the manager of a firm follows the pursuit of happiness rather than the pursuit of benefits. As we explained in the introduction, it is reasonable to assume that bankruptcy is costly for a manager-entrepreneur; this fact leads Grossman and Hart (1982) to argue that issuing debt is a pre-commitment or bonding behavior aimed at convincing investors that the firm will be managed to maximize profits, so as to avoid bankruptcy. Hence, a high leverage could signal the good prospects of the firm and be the guarantee that investments will be carried on at the efficient level.

To check formally this claim, we consider first the manager of a firm that is a distinct from the owners. The firm (aka. the owner) raises an amount \( F \) of funds by selling a mix of equity and a debt obligation \( d \). The moral hazard issue here is the fact that it is the manager who allocates \( k \) into physical capital and the remnant \( F - k \) into her human capital which we interpret as a private benefit since it is an unalienable asset that cannot be taken away by the owners.\(^{28}\) What disciplines the manager is the fact that she will enjoy her private benefits only if she remains at the head of the firm.

The project’s future cash flow is \( R(k) + \bar{x} \) where \( \bar{x} \) is a random shock of zero

\(^{28}\)The remnant could also be more brutally diverted into perquisites i.e., an immediate consumption of no value for the firm.
mean\(^{29}\) so that the expected NPV of profits is \(R(k) - k\). Given the uncertainty regarding the future cash flow, the firm will go bankrupt whenever \(R(k) + \tilde{x} < d\), hence the expected utility of the manager is the average of her perks \(F - k\) over the states of nature where she enjoys them i.e.,

\[
\mathbb{E} [F - k|R(k) + \tilde{x} \geq d] = (F - k) (1 - H(d - R(k)))
\]

The optimal investment \(\hat{k}\) solves the FOC

\[
(F - k) R_m(k) h(d - R(k)) = 1 - H(d - R(k)) \Leftrightarrow \frac{1}{(F - k) R_m(k)} = \phi(d - R(k))
\]

where \(\phi(x) \equiv \frac{h(x)}{1 - H(x)}\) is the hazard rate of the distribution function \(H\).\(^{30}\) In equilibrium of the capital market, investors anticipate the choice \(\hat{k}\), thus value the firm's securities \(F\) as the present value \(R(\hat{k})\) of the project; the first order condition determining \(\hat{k}\) then becomes \(\frac{1}{(R(k) - k) R_m(k)} = \phi(d - R(k))\) and we observe on Figure 5.4 that an increase in debt from \(d_1\) to \(d_2\) moves the RHS up, hence the optimal investment increases from \(k_1\) to \(k_2\); we can therefore conclude that the leverage chosen by the entrepreneur is a commitment to invest mostly into the firm's future value and not into her personal satisfaction.

Adding a greater degree of realism is possible without changing the qualitative nature of the result. Firstly, the utility for the manager need not be linear in the perks she keeps for herself, it can be \(u(F - k)\) for a concave increasing utility function. Then the FOC becomes \(\frac{u'(F - k)}{u(F - k) R_m(k)} = \phi(d - R(k))\) but since the ratio \(\frac{u'(x)}{u(x)}\) is a decreasing function of \(x\), our previous conclusions remain unchanged. Then we are able to consider the case of an entrepreneur instead of a manager; in that case the utility she maximizes is \(\mathbb{E}[u(F - k) + R(k) + \tilde{x} - d|R(k) + \tilde{x} \geq d]\) so that the FOC becomes \(\frac{u'(F - k) - R_m(k)}{u(F - k) R_m(k)} = \phi(d - R(k))\) and once again, the qualitative effect of raising the leverage remains a commitment to later choose a greater investment.

\(^{29}\)Instead of an additive uncertainty, we could equally use a multiplicative market price uncertainty like in previous models at the cost of heavier formulas.

\(^{30}\)We assume that \(H\) is one of the many distributions whose hazard rate is increasing. Since the LHS of the FOC is increasing with \(k\) while the RHS is decreasing, there is indeed a unique solution \(\hat{k}\).
5.3.3 Efficiency Wage

Intuition

The persistence of unemployment in competitive economies has long been a puzzle that standard neoclassical theory could not solve. Indeed, in the many economies around the world that do not guarantee a minimum wage, why can't the market wage descend low enough (with subsistence as a lower bound) to stimulate demand and thereby provide a job to every person who seeks one? Shapiro and Stiglitz (1984) offer an innovative explanation based on incentives and asymmetry of information.

To understand their theory it is useful to work by contradiction. If there existed an equilibrium with full employment, a fired worker would be instantaneously rehired at the same equilibrium market wage. Thus, in terms of utility, she would barely notice the change which means, adopting the firm's point of view, that there is no way to penalize a worker caught shirking (not working as hard as stipulated in her contract). Thus, to motivate hard work, firms are forced to pay above-market wages in order that the loss of one's job be painful. But, as always in economy, if one firm finds it attractive to pay above-market wages, then all firms will do the same. This means that the market wage increases and exactly matches the wage paid by each firm, hence work incentives have been destroyed once again. However, wages being above their natural equilibrium values, labor has become a more expensive input (lower productivity) and its ag-
aggregate demand falls generating *unemployment*. Now, the consequence of being fired (for being caught shirking) is more dreadful than before because the individual will have to wait before finding a new job and will have to live poorly in the meantime.

We have thus seen that the wage is not only the price of the labor input equating demand and supply but also a device to provide work incentives inside the firm. As intuition suggests, when one instrument is used to solve two problems, some inefficiencies are bound to appear.

**Model**

To check this claim, consider identical risk neutral workers whose utility is $w - q$ where $w$ is the wage and $q$ the effort they exert (0 if jobless). The common rate of discount is $r$. For simplicity, the efficient effort is a fixed level $q > 0$. At each period, a person can be unemployed (type $u$) or enjoy a job (type $j$) in which case he will either work obediently ($o$) or shirk ($s$). Jobless people receive a benefit $b$ that can be provided by the State or family. There is an exogenous probability $\rho$ of losing one’s job (e.g., the firm goes bankrupt) and a probability $\lambda$ of being caught by the monitoring technology if shirking.

The value of being employed and unemployed are denoted $V_j$ and $V_u$; the present value of remaining in a given situation forever is $V \times \sum_{k \geq 1} \frac{1}{(1+r)^k} = rV$. For an employed “shirker” with present value $V_j^s$ we also have

$$V_j^s = w - (\rho + \lambda)(V_j^s - V_u) \tag{5.18}$$

i.e., what she expects is equal to her current wage minus the expected loss of utility relative to the possible job termination that might occur at the end of the period (if caught shirking). For an “obedient” worker, the effort must be accounted while the probability of losing one’s job is lower, thus the equation is

$$rV_j^o = w - q - \rho(V_j^o - V_u) \tag{5.19}$$

Solving the two equations so obtained we derive

$$V_j^s = \frac{w + (\rho + \lambda)V_u}{\rho + \lambda + r} \quad \text{and} \quad V_j^o = \frac{w - q + \rho V_u}{\rho + r} \tag{5.20}$$
Shirking won’t take place only if \( V_j^o > V_j^s \Leftrightarrow w > \hat{w} \equiv r V_u + \frac{(\rho + \lambda + r)q}{\lambda} \). We can now assume that employers will pay exactly \( \hat{w} \) to avoid shirking by their employees.

For a jobless person, there is a unique equation since he does not have to decide if he shall work or shirk. Letting \( \alpha \) denote the job acquisition rate, the value of being unemployed satisfies

\[
r V_u = b + \alpha (V_j - V_u) \tag{5.21}
\]

Since in equilibrium, workers do not shirk we have \( V_j = V_j^o \); (5.20) and (5.21) form a system whose solution is

\[
V_j = \frac{(\hat{w} - q)(\alpha + r) + b \rho}{\alpha + \rho + r} \quad \text{and} \quad V_u = \frac{(\hat{w} - q)\alpha + b(\rho + r)}{\alpha + \rho + r} \tag{5.22}
\]

Replacing the latter formula into the definition of \( \hat{w} \) further yields \( \hat{w} = b + q + \frac{(\alpha + \rho + r)q}{\lambda} \). In terms of comparative statics the critical wage must be larger when work is more painful (larger \( q \)), when the unemployment benefit \( b \) is larger, when the probability of being caught shirking \( \lambda \) is lower, when the interest rate \( r \) is larger (preference for the present) and when the economy is more unstable (larger \( \rho \)).

Lastly, using the total worker population \( N \) and the employed population \( L \), we can derive \( \alpha \) since in equilibrium, the flows out of unemployment equal the flows in so that \( \alpha (N - L) = \rho L \Rightarrow \alpha = \frac{\rho L}{N - L} \) and \( \hat{w} = b + q + \frac{q}{\lambda} \left( \frac{\rho N}{N - L} + r \right) \). As we can see on Figure 5.5, this relation draws a frontier \( L = S(w) \) between combinations of employment and wages that induce shirking or work inside firms. We can interpret this curve as a labor supply function.

To derive the equilibrium on the labour market we have to consider the behavior of firms; we only need to assume that the marginal productivity of labour is decreasing with employment to obtain a downward sloping demand curve \( D \). Also the productivity of the fixed effort \( q \) must be large enough to warrant full employment at the complete information equilibrium i.e., \( D(b + q) > N \) as represented on Figure 5.5. Given the moral hazard issue of shirking, firms are forced to add the incentive constraint \( L \leq S(w) \) to the traditional technological constraint \( L \leq D(w) \) i.e., they limit their demand schedule to the upper part of the \( D \) curve so that the equilibrium is a pair \((w^*, L^*)\) generating unemployment.

Some interesting observations are:
All the above comparative statics regarding conditions that increase the critical wage turn out to increase unemployment.

Jobless workers would accept a lower wage but since they are unable to commit not to shirk, firms prefer to let them out.

During a recession the labor demand moves down which lowers wages but the probability of shirking rises (larger $\rho$) so that unemployment is doubtlessly increased.

The equilibrium is inefficient because firms employ too few (they equate $w^*$ not $e$ to their productivity) and because each of them causes a negative externality on others given that $V_u$ is increased for other firms when one of them hires more people.

If the workers are the communist owners of firms then the optimal level of employment maximizes workers utility $(w - q)L + b(N - L)$ under the “no shirking” constraint $w \geq \hat{w}$ and the industry participation constraint $wL \leq \Phi(L)$. Given that $w \geq \hat{w} \Rightarrow w > b + q$, the social objective turns out to be the maximization of employment under the two previous constraints i.e., the optimum solves $\hat{w}(L) = \Phi(L)/L$. For most industries, there are diseconomies of scale so that the average productivity is greater than the marginal one and leads to a greater employment than at the market equilibrium. If workers and owners are different economic agents then the only change in the above Pareto program is $wL \leq \Phi(L) + \theta$ where $\theta \geq 0$ is a non negative parameter. As a result, optimal
employment is reduced but it still remains above the market equilibrium level. Hence we may conclude that asymmetric information and costly monitoring generate a market failure creating too much unemployment. The government is thus warranted to intervene to reduce unemployment by using (not too distorting) taxes.
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